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A PRIORI IDENTIFICATION OF THE MODE OF CONVERSION FROM AN IMPERFECTION-SENSITIVE INTO AN IMPERFECTION-INSENSITIVE ELASTIC STRUCTURE

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ABSTRACT

Rather than being concerned of how to properly deal with imperfection sensitivity of a structure, i.e., pondering over suitable estimates of unknown imperfections and over the necessity of a dynamic postbuckling analysis, it is worth while to consider the possibility of conversion from an imperfection-sensitive into an imperfection-insensitive elastic structure by means of minor structural modifications. Admittedly, architectural and/or functional constraints may not always allow such modifications of the original design of a structure. Moreover, the mentioned conversion may be incomplete because of not including all relevant load cases.

Assuming, however, that a meaningful conversion from an imperfection-sensitive into an imperfection-insensitive structure is feasible, it is useful to have *a priori* knowledge of the mode of this conversion which defines the quality of the postbuckling behavior of the imperfection-insensitive-turned perfect structure.

Having a strong influence on the behavior of the imperfect structure, this quality can be assessed by means of the asymptotic series expansions [1]

$$\lambda(\eta) = \lambda_C + \lambda_1\eta + \lambda_2\eta^2 + \lambda_3\eta^3 + \lambda_4\eta^4 + O(\eta^5), \tag{1}$$

$$\boldsymbol{v}(\eta) = \qquad \boldsymbol{v}_1\eta + \boldsymbol{v}_2\eta^2 + \boldsymbol{v}_3\eta^3 + \boldsymbol{v}_4\eta^4 + O(\eta^5), \tag{2}$$

where $\lambda(\eta)$ is the load parameter of a point on the secondary path, defined by the path parameter η. The point on the primary path defined by the same load is described by the displacement vector $\tilde{\mathbf{u}}(\lambda(\eta))$. The displacement at the corresponding point on the secondary path can be written as $\mathbf{u}(\eta) = \tilde{\mathbf{u}}(\lambda(\eta)) + \mathbf{v}(\eta),$ where $\mathbf{v}(\eta)$ is the displacement offset.

Setting $\lambda_1 = 0$, as is required for imperfection insensitivity, the following expression can be derived for λ_4 [1]:

$$\lambda_4 = a_1\lambda_2^2 + b_2\lambda_2 + d_3, \tag{3}$$

where a_1, b_2, and d_3 are coefficients depending on $\lambda_C(\kappa)$, with κ denoting a design parameter such as e.g. the stiffness of an elastic spring attached to the structure. Remarkably, for $\lambda_1 = 0$, λ_4 does not depend on λ_3 [1].

At the transition from imperfection sensitivity to insensitivity in the course of increasing the value of κ,

$$\lambda_2 = 0 \quad \xrightarrow{(3)} \quad \lambda_4 = d_3, \tag{4}$$

where λ_4 may be positive, negative, or zero. For $\kappa \to \infty$, characterizing the practically meaningless limiting value of the design parameter,

$$\lambda_2 \to +\infty, \quad \lambda_4 \to +\infty. \tag{5}$$

Alternatively,

$$\lambda_4 = -a_1 \lambda_2^2 + d_3, \tag{6}$$

which requires

$$2a_1 \lambda_2 + b_2 = 0. \tag{7}$$

At $\lambda_2 = 0,\ \lambda_4 = d_3 < 0.$ In contrast to (4), for $\kappa \to \infty$,

$$\lambda_2 \to +\infty, \quad \lambda_4 \to -\infty. \tag{8}$$

Hence, the asymptotic behavior associated with this mode of conversion from imperfection sensitivity into imperfection insensitivity differs from the one associated with the previous mode of such a conversion. More importantly, the quality of the imperfection insensitivity encountered after the conversion from imperfection sensitivity is worse than the one associated with the previous mode of conversion. Typically, the secondary path exhibits a snap-through point, which may have a negative influence on the behavior of the imperfect structure.

The correlation of (3) and (6) with two different modes of specialization of the expression for $\boldsymbol{v}^*_{1,\lambda\lambda}$, denoting the second derivative of the eigenvector of the so-called consistently linearized eigenproblem [2] with respect to the load parameter, will also be shown.

The theoretical findings will be corroborated by numerical examples.

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