8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A NON-LINEAR TRANSFORMATION FOR THE ACCURATE INTEGRATION OF ANISOTROPIC MICROSPHERE-BASED MATERIAL MODELS WITH APPLICATION TO BLOOD VESSELS MODELLING

*** V. Alastrué[1], M. A. Martínez[1], M. Doblaré[1], and A. Menzel[2,3]**

[1] Group of Structural Mechanics and Materials Modelling. Aragón Institute of Engineering Research (I3A), University of Zaragoza. CIBER-BBN. Networking Centre on Bioengineering, Biomaterials and Nanomedicine. Aragón Institute of Health Sciences.
María de Luna, 3. E-50018 Zaragoza, Spain.
{victorav,miguelam,mdoblare}@unizar.es

[2] Department of Mechanical Engineering. Mechanics and Machine Dynamics, University of Dortmund.
Leonhard-Euler-Str. 5, D-44227 Dortmund, Germany.
andreas.menzel@udo.edu

[3] Division of Solid Mechanics, Lund University.
P.O. Box 118, SE-22100 Lund, Sweden.
andreas.menzel@solid.lth.se

Key Words: *Microsphere, Anisotropy, Non-Linear Transformation, Blood Vessels.*

ABSTRACT

During the last years, the constitutive modelling of soft biological tissues has constituted a very active field of research. Usually, they are composed of an extra-cellular matrix formed by an isotropic high water content ground substance in which a network of elastin and collagen fibres is embedded [1]. These fibres render the behaviour of the tissue to be highly non-linear under finite strains. Commonly, these materials have been modelled as hyperelastic continua embedded into continuum mechanical formulations, whereas inclusion of structural tensors into constitutive laws is the most widely used technique to introduce anisotropy caused by fibres [3,6]. Nevertheless, the large variability concerning the mechanical behaviour and the particular composition exhibited by soft biological tissues requires the incorporation of representative structural information into associated constitutive models in order to appropriately reproduce the response of these materials.

Microsphere-based models incorporate the individual response of the underlying material constituents so that the macroscopic behaviour is obtained by means of a computational homogenisation technique [5]. Incorporation of anisotropy present in blood vessels in microsphere-based models is achieved by means of the inclusion of a orientation density function (ODF) that weights the contribution of the micro-fibres in each direction of space respect to a preferred orientation direction $\boldsymbol{a}$ [4]. Then, the

macroscopic energy turns out

$$\Psi = \langle n\,\rho\,\Psi_f \rangle := \frac{1}{4\,\pi} \int_{\mathbb{U}^2} n\,\rho\,\Psi_f\,\mathrm{d}A \approx \sum_{i=1}^{m} w^i\,\rho^i \Psi_f^i, \tag{1}$$

where n is the micro-fibres density, ρ is the named ODF, and Ψ_f represents the mechanical response of an individual micro-fibre. Normalization the integral over the surface of the unit sphere is done by means of the factor $A_{\mathbb{U}^2} = 4\,\pi$. Discretisation of Eq. (1) is represented by its rightmost term, in which w^i represent weighting factors. To compute stress-free states, the conditions $\langle \boldsymbol{r} \rangle \approx \sum_{i=1}^{m} w^i\,\boldsymbol{r}^i = \boldsymbol{0}$ as well as $\langle \boldsymbol{r}^i \otimes \boldsymbol{r}^i \rangle \approx \sum_{i=1}^{m} w^i\,\boldsymbol{r}^i \otimes \boldsymbol{r}^i = \frac{1}{3}\boldsymbol{I}$ constraint Eq. (1), being $\{\boldsymbol{r}^i\}_{i=1,\cdots,m}$ discrete unit orientation vectors.

Among the multiple ODFs used to model blood vessels behaviour, the π-periodic von Mises distribution in Eq. (2) was used [2].

$$\rho(\arccos{(\boldsymbol{r} \cdot \boldsymbol{a})}) = \rho(\theta) = 4\sqrt{\frac{b}{2\pi}}\,\frac{\exp{(b\,[\cos(2\,\theta) + 1])}}{\mathrm{erfi}(\sqrt{2\,b})}, \tag{2}$$

where the positive concentration parameter b constitutes a measure of the degree of anisotropy, and $\mathrm{erfi}(x) = -\,i\sqrt{\frac{2}{\pi}} \int_0^x \exp(-\,\xi^2)\,\mathrm{d}\xi$ denotes the imaginary error function. Note that the isotropic case is represented by $b = 0$, whereas transversal isotropy is reached for $b \to \infty$.

Within a computational framework, it has been demonstrated that a $m = 42$ directions integration scheme produces accurate results for the isotropic case [5], whereas more integration directions are needed in order to accurately integrate Eq. 1. To give an extreme example, the purely transversely isotropic case would, from the computational perspective, render a non-existent contribution unless the mean direction $\boldsymbol{a}$ coincides with one integration direction. Therefore, highly anisotropic materials will require extremely fine discretizations of the microsphere, which results in a notable increase of the computation time. A non-linear transformation of the microsphere discretisation is here proposed so that it is possible to reduce the number of integration directions for highly anisotropic cases. Redistribution of integration direction around the anisotropy preferred orientation turns out in faster and simpler integration schemes for microsphere-based constitutive models.

REFERENCES

[1] Y. C. Fung. *Biomechanics: Mechanical Properties of Living Tissues*. Springer, 1990.

[2] T. C. Gasser, R. W. Ogden, and G. A. Holzapfel. “Hyperelastic modelling of arterial layers with distributed collagen fibre orientations.” *J Roy Soc Interface*, 3:15–35, 2006.

[3] G. A. Holzapfel, T. C. Gasser, and R. W. Ogden. “ A new constitutive framework for arterial wall mechanics and a comparative study of material models. ” *J Elasticity*, V61(1):1–48, July 2000.

[4] Y. Lanir. “ A structural theory for the homogeneous biaxial stress-strain relationships in flat collagenous tissues. ” *J Biomech*, 12(6):423–436, 1979.

[5] C. Miehe, S. Göktepe, and F. Lulei. “ A micro-macro approach to rubber-like materials–part i: the non-affine micro-sphere model of rubber elasticity. ” *J Mech Phys Solids*, 52(11):2617–2660, November 2004.

[6] J. A. Weiss, B. N. Maker, and S. Govindjee. “ Finite element implementation of incompressible, transversely isotropic hyperelasticity. ” *Comput Method Appl M*, 135(1-2):107–128, August 1996.