An inverse elliptic source problem from boundary measurements

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Abstract

Inverse problems are very important in science, engineering and bioengineering. Among these, inverse source problems have attracted great attention of many researchers over recent years because of their applications to many practical examples. Among these applications we quote two for which there is abundant literature: identification of pollution sources in environment and current dipolar sources in so-called inverse electroencephalography/magnetoencephalography (EEG/MEG) problems. Many theoretical and numerical studies using iterative algorithms that seek to minimize the error between the observable data and the solution of the forward problem or direct algorithms for obtaining source parameters have been studied. Direct algorithms require algebraic relations between source parameters and observable data.

In this talk we consider an inverse source problem for an anisotropic elliptic equation for which the source is a combination of monopolar and dipolar sources.

Let $\Omega \subset \mathbb{R}^n, n = 2, 3$ be a open bounded domain with a sufficiently regular boundary Γ. In this talk we are interested in determining a source term F, from boundary data f, in the following anisotropic elliptic equation

$$-\nabla.(\gamma \nabla u) = F \text{ in } \Omega$$

with the Neumann boundary condition

$$\nu.\gamma \nabla u = g \text{ on } \Gamma.$$

Here ν represents the outward unit normal vector to Γ.

For this purpose let $\gamma = (\gamma_{ij})$ denote the anisotropic conductivity of the medium Ω and u the steady state electric potential associated to the "current flux" g and the source F. The coefficients γ_{ij} are constant and the $n \times n$ matrix γ is assumed to be symmetric positive definite.

Moreover, we assume the "current flux" $g \in H^{-\frac{1}{2}}(\Gamma)$ and let the source F be of the form

$$F = \sum_{k=1}^{m_1} \lambda_k \delta_{C_k} + \sum_{j=1}^{m_2} \mathbf{q}_j . \nabla \delta_{S_j}$$

where m_1, m_2 are positive integers, C_k and S_j are points in Ω, λ_k and $\mathbf{q}_j$ are respectively, scalar and vector quantities such that $\lambda_k \neq 0$, $\mathbf{q}_j \neq 0$. Furthermore, the points C_k, S_j are assumed to be distinct.

One of the possible applications, but not the only one, of our study can be the inverse source problem of locating epileptic foci in the human brain, the so-called inverse electroencephalography problem corresponding to the case $\lambda_k = 0$. That is the so-called dipolar sources.

In this talk four questions will be studied, identifiability, local Lipschitz stability, identification and global stability using a single boundary measurement. We prove an identifiability result and establish a local Lipschitz stability result for linear combination of monopolar and dipolar sources when the measured data is available on only a part of the boundary. Unfortunately our identification method and global stability do not apply to combined monopolar and dipolar sources, so we consider only the dipolar sources for which the study is more difficult than for monopolar sources. Moreover they require the knowledge of the measurement data on the whole boundary.

References

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