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# BOUNDARY ELEMENT METHOD FOR MICROPOLAR FLUID FLOW MODELING IN AN ENCLOSURE

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## ABSTRACT

Flows in the nature are almost always formed from multiphases. Such a fluid flows are very complex and express different behaviour under different conditions. Therefore we are interested to use proper numerical models to describe physical behaviour of such a fluid flow. In suspension flows modeling where we consider that particles are rigid and underformable with own rotation, we can use micropolar fluid flow theory. This theory also enables accurate computation of flows in a scale, where questions arise on the accuracy of the Navier-Stokes equation. Micropolar fluids are subclass of microfluids introduced by Eringen [1]. Simple microfluid is by Eringen’s definition fluent medium whose properties and behaviour are influenced by the local motions of the material particles contained in each of its volume element. A microfluid is isotropic, viscous fluids and possesses local inertia. As mentioned, in micropolar fluids, which are subclass of microfluid, rigid particles contained in a small volume element can rotate about the center of the volume element described by the micro-rotation vector Eringen [2]. This local rotation of the particles is independent of the mean fluid flow and its local vorticity field. From this theory is also expected to describe successfully non-Newtonian behaviour of certain fluids, such a liquid crystals, ferro liquids, colloidal fluids and liquid with polymer additives.
Among different approximation methods for solving problems of fluid flow Boundary Element Method (BEM) is increasingly gaining attention. In this work, the micropolar fluid flow theory is incorporated into the framework of velocity-vorticity formulation of Navier-Stokes equations presented by Škerget et al. [3] and show how to incorporate the micropolar fluid theory into the BEM framework. Governing equations are derived in differential (eq. 1-4) as well as integral form.

$$\frac{\partial^2 v_i}{\partial x_j \partial x_j} + e_{ij}\frac{\partial \omega}{\partial x_j} = 0 \tag{1}$$

$$\rho\frac{D\omega}{Dt}=(\mu_v+k_v)\frac{\partial^2\omega}{\partial x_j\partial x_j}-k_v\frac{\partial^2 N}{\partial x_j\partial x_j}-\beta_T\frac{\partial g_i(T-T_o)}{\partial x_j} \quad (2)$$

$$c_p\rho\frac{DT}{Dt}=\lambda\frac{\partial^2 T}{\partial x_j\partial x_j} \quad (3)$$

$$\rho j\frac{DN}{Dt}=\gamma_v\frac{\partial^2 N}{\partial x_j\partial x_j}+k_v e_{ij}\frac{\partial v_i}{\partial x_j}-2k_v N \quad (4)$$

Diferential operator $D(\cdot)/Dt=\partial(\cdot)/\partial t+v_k\partial(\cdot)/\partial x_k$ represents the Stokes material derivative. In eq. 1-4 is presenting $v$ velocity, $\omega$ vorticity, $T$ temperature, $N$ microrotation, $\rho$ fluid mass density, $\beta_T$ thermal expansion coefficient, $c_p$ specific isobaric heat per unit mass, $\lambda$ conduction, $\mu_v$ dynamic viscosity, $k_v$ vortex viscosity coefficient, $\alpha_v,\beta_v,\gamma_v$ viscosity gradient coefficients and $j$ microinertia. Derived numerical algorithm is verified on example of natural convection in partial heated enclosure. Natural convection is a physical phenomenon, where in presence of the temperature difference between body surfaces buoyancy differences appeared. Most fluids near a hot wall will have their density decreased, and an upward near wall motion will be induced. Natural convection of micropolar fluid in rectangular enclosure was presented in the papers from Aydin and Pop [4] for different Rayleigh and Prandtl numbers. In the table 1 the results for $k_v=0$ (newtonian fluid) are compared with benchmark results of newtonian fluid from Davies [5] and with micropolar fluid flow results from Aydin and Pop [4] also for $k_v=0$.

Table 1: Comparison of averaged Nusselt number in dependence of Rayleigh number.

| Ra | *Present study* | *Davies [5]* | *Aydin and Pop [4]* |
|---|---|---|---|
| $10^3$ | 1,118 | 1,118 | 1,118 |
| $10^4$ | 2,263 | 2,243 | 2,234 |
| $10^5$ | 4,54 | 4,519 | 4,486 |
| $10^6$ | 8,742 | 8,8 | 8,945 |
| $10^7$ | 17,323 | --- | --- |

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