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ON NUMERICAL MODELLING OF CREEP BEHAVIOUR OF MEDIUM DENSITY POLYETHYLENE

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ABSTRACT

The increasing application of polymeric materials especial polyethylenes as structural materials demands new methodologies in order to assess the material capability to withstand loads. The use of medium density polyethylene pipes for water and gas distribution is one of the most common examples. An accurate modeling of fracture and viscoelastic material responses of such structures, represent a key for the prediction of structural integrity.

Since the polyethylene structures are mostly subjected to creep loadings, the present paper is concerned with the numerical modeling of creep fracture mechanisms by slow crack growth [1]. The finite element code ABAQUS [2] is used for analyzing the creep deformation and the fracture parameters from small-scale creep to steady-state creep conditions. The material under study is assumed to obey the following primary-secondary creep constitutive law at 60 and 80 ºC:

$$\dot{\varepsilon}_{\mathrm{c}} = B_1\, p\, t^{(p-1)}\, \sigma^{n_1} + B_2 \cdot \sigma^{n_2}\,, \tag{1}$$

where σ and ε_{c} are the stress and creep strain, while B_1, B_2, p, n_1 and n_2 stand for the material coefficients. Dot designates derivative with time t. The first term on the right-hand side of Eq. (1) represents the primary creep and the second term is the secondary portions of creep deformation. The viscoelastic material model is taken from the literature [3] and the constitutive laws are implemented in the code ABAQUS by using a user subroutine CREEP. The computational strategy is based on the time hardening integration approach [2, 4]. Therein, a detailed analysis is performed to evaluate correct value of the strain energy density rate function, $\dot{W}$, defined as

$$\dot{W} = \int_0^{\dot{\varepsilon}_{\mathrm{c}}} \sigma\, \mathrm{d}\dot{\varepsilon}_{\mathrm{c}}\,, \tag{2}$$

and numerical validation is given [5]. Additionally, for the implicit integration algorithm the Jacobian matrix is derived and applied thereby.

Thereafter systematic detailed non-linear finite element analyses have been carried out to determine the *C*-integral as a function of time for three different defective components. Analyses are performed on axisymmetrically cracked specimens denoted as full notched creep tensile, on double edge notched tensile specimens as well as on tubes under internal pressure. In order to achieve high numerical efficiency a combined explicit and implicit integration scheme is used for the creep calculations. Accuracy and robustness of the proposed algorithms are demonstrated. The values of the *C*-integral and time to fracture are compared with the published empirical solutions [3].

Using numerical solutions a new analytical approximation of the *C*-integral has been derived which is applicable to a wide range of crack dimensions. Based on the analogy between plasticity and creep, the C^*-integral for steady-state creep conditions is formulated using the reference stress approach introduced by Ainsworth [6]

$$C^* = \left(\frac{K^2}{E'} \right) \frac{E \dot{\varepsilon}_c}{\sigma_{\text{ref}}}, \tag{3}$$

where K is the elastic stress intensity factor, $E' = E\left(1-\nu^2\right)$ for plane strain and $E' = E$ for plane stress and σ_{ref} denotes the reference stress which is expressed as

$$\sigma_{\text{ref}} = \sigma_{\text{y}} \frac{P}{P_{\text{RL}}}. \tag{4}$$

In Eq. (4) σ_{y} is the yield stress, while P_{RL} represents the reference load. It is shown that the proposed analytical approximation of C^*-integral provides very useful tools for the assessing of polyethylene components integrity.

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