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SOLUTION OF CONTACT PROBLEMS WITH LARGE DISPLACEMENTS AND DEFORMATIONS USING P-VERSION FINITE ELEMENTS

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ABSTRACT

An axially symmetric model of an air-spring is analyzed numerically by FEM. The elastic part of the spring is a fiber reinforced rubber composite. The problem is strongly nonlinear due to incompressibility of the rubber, large displacements, large deformations and unilateral contact. The analysis is performed by p-version of the FEM.

In our investigation the rubber ($\beta = 1$) is assumed to be a nearly incompressible material and it is modeled with the Hu-Washizu functional, see [1], and the fiber reinforced layer ($\beta = 0$) is homogenized by the so called Halpin-Tsai equations

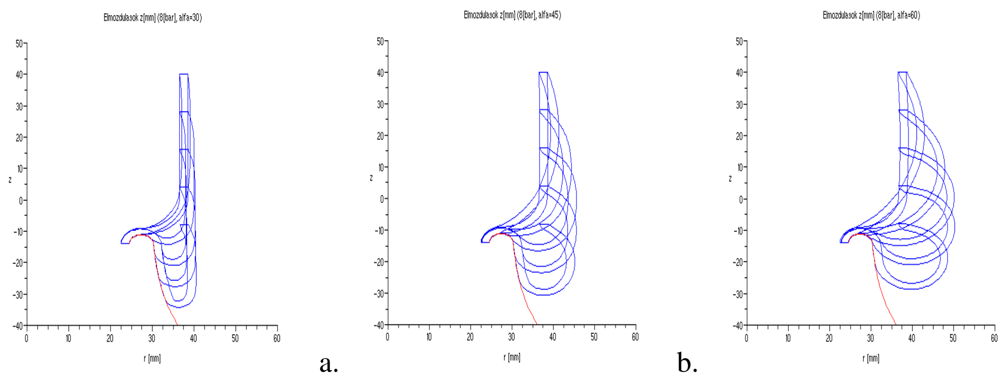
$$\Pi_{HW}\left(\mathbf{u},\overline{J},\overline{p}\right) = \beta\left\{\begin{matrix} \int\limits_V \widehat{W}(\mathbf{C})\,dV + \int\limits_V U\left(\overline{J}\right)dV \\ +\int\limits_V \overline{p}\left(J-\overline{J}\right)dV - \int\limits_{S_t} \mathbf{u}\cdot\mathbf{n}\tilde{p}\,dA \end{matrix}\right\} + (1-\beta)\frac{1}{2}\int\limits_V \mathbf{E}\cdot\cdot\mathbf{D}\cdot\cdot\mathbf{E}\,dV\,,$$

where ' $\cdot\cdot$ ' means the double dot product of two tensors, $\mathbf{u}$ is the displacement field, $J,\overline{J}$ is the determinant of the deformation gradient tensor and the field of the volumetric change, respectively, $\overline{p}$ is the hydrostatic pressure, $\tilde{p}$ is the prescribed pressure on the boundary S_t, $\widehat{W}$ is the Mooney-Rivlin strain energy density, $\mathbf{C}$ is the right Cauchy-Green tensor, $U\left(\overline{J}\right) = \frac{\kappa}{50}\left(\overline{J}^5 + \overline{J}^{-5} - 2\right)$ is the penalty function proposed by [2], $\mathbf{E}$ is the Green-Lagrange strain tensor, $\mathbf{D}$ is the constitutive tensor of the orthotropic fiber reinforced layer. The model consists two fiber reinforced layers with opposite orientations.
The displacements are approximated by tensor product space of the Legendre functions [3] ensuring C^0 continuity, the hydrostatic pressure and the volumetric change are approximated one order lower polynomial functions independently element by element.

The contact problem is treated by a simplified approach. The contacting boundary is approximated by a polygon, i.e. the edge of the contacting element is enforced to be a straight line also when high order displacement approximation is used. Practically three-node penalty contact elements were implemented, detailed in the book [4]. The contact problems are solved considering the Coulomb frictional effect, the wear process is also taken into account.

Assuming the isotropic wear rule in the form $\dot{w} = \beta\,(\mu\, p_n)^b\, v_r^a = \tilde{\beta}\; p_n{}^b\, v_r^a$, where a, b, β are wear parameters, $\mu,\, p_n$ are coefficient friction and contact pressure, $\tilde{\beta} = \beta\,\mu^b$, $v_r = \|\dot{u}_\tau\|$ is the relative velocity between the bodies. The shearing stress in the contact surface is calculated from the contact pressure by the Coulomb dry friction law $\tau_n = \mu\, p_n$. The wear between the rubber and metal surface can be defined by cyclic loadings after time integration, $w = \int_0^{T_*} \dot{w}\, d\tau$, where T_* is the time of observation.

The contact problems are analyzed numerically for different fiber orientations ($\alpha = \pm 30^o$, $\alpha = \pm 45^o$, $\alpha = \pm 60^o$) and frictional coefficients assuming quasi static loadings. Since the contact deformations and contact pressure distributions are strongly depend on the fiber orientations the wear process essentially will be also different.



a. b. c.

Figure 1

Deformed shapes of the air-spring in case of different values of α:

a. $\alpha = \pm 30^o$, b. $\alpha = \pm 45^o$, c. $\alpha = \pm 60^o$

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