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ACCURATE STRESS RECOVERY IN CONTACT PROBLEMS

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ABSTRACT

The accurate evaluation of normal and tangential stresses in contact problems is required to warrant the convergence of the algorithms used to solve contact problems as the behaviour of these algorithms highly depends on the accuracy of the contact forces.

In this paper we propose an adaptation of the SPR-C technique[1] (*constrained* SPR) for the accurate evaluation of stresses in contact problems. The SPR-C technique is an enhanced version of the Superconvergent Patch Recovery (SPR) technique [2] which uses the appropriate constraint equations to obtain stress interpolation polynomials in the patch, $\boldsymbol{\sigma}_p^*$, that locally satisfy the equations that should be satisfied by the exact solution (internal and boundary equilibrium equations and the compatibility equation). The SPR-C technique, that was first used for the resolution of linear elasticity problems[1], has been adapted to account for the stress relations that arise between the bodies in contact, i.e. continuity in normal and tangential stresses.

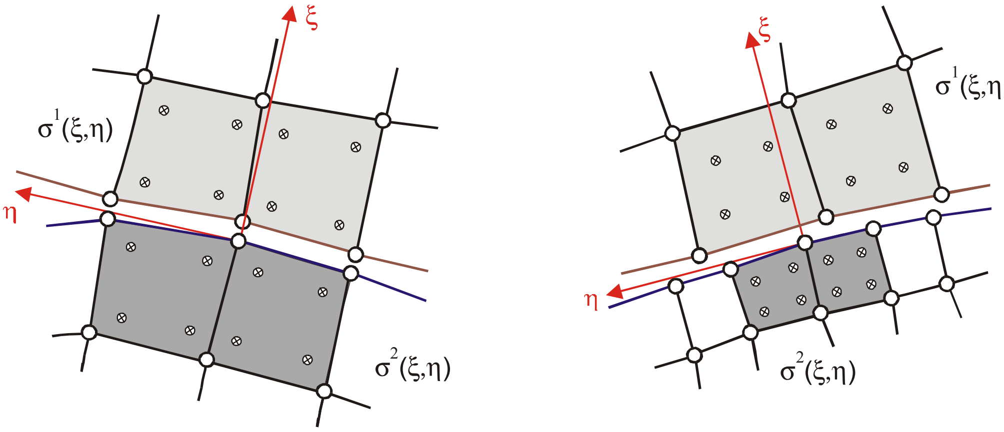

Figure 1.- Patch generation in contact nodes in conforming and non-conforming meshes

The adaptation of the SPR-C technique to the contact problems allowed us to simultaneously take into account the stress fields of the two bodies in contact. Note that although the stress components are continuous, different stress interpolation polynomials must be used at each body. When the patch corresponding to a node n_{c1}

located in the contact area is created, a second patch is also created with the elements attached to node n_{c2}, which is the closest node to n_{c1} in the second body, as represented in Figure 1. The stress interpolation polynomials used to obtain the recovered stresses in each of the two bodies (α=1,2),can be written as:

$$\sigma_i^{\alpha}(\xi,\eta)=a_{i1}^{\alpha}+a_{i2}^{\alpha}\xi+a_{i3}^{\alpha}\eta+a_{i4}^{\alpha}\xi^2+a_{i5}^{\alpha}\xi\eta+a_{i6}^{\alpha}\eta^2+\ldots \qquad i=\xi,\eta,\xi\eta \qquad \alpha=1,2 \tag{1}$$

The following constraints are used to enforce stress continuity along the contact area $(\xi=0)$

$$\text{Normal stress continuity:} \qquad a_{\xi 1}^{1}=a_{\xi 1}^{2}; \qquad a_{\xi 3}^{1}=a_{\xi 3}^{2}; \qquad a_{\xi 6}^{1}=a_{\xi 6}^{2} \tag{2}$$

$$\text{Tangential stress continuity:} \qquad a_{\xi\eta 1}^{1}=a_{\xi\eta 1}^{2}; \qquad a_{\xi\eta 3}^{1}=a_{\xi\eta 3}^{2}; \qquad a_{\xi\eta 6}^{1}=a_{\xi\eta 6}^{2} \tag{3}$$

The unknown coefficients a_i^{α} are obtained by simultaneously solving the systems of equations corresponding to the two patches in contact, considering the constraints in (2) and (3) and those necessary for the satisfaction of the internal equilibrium and compatibility equations as described in [1].

The comparison of the error in the recovered stresses along the contact area obtained with the SPR and SPR-C techniques is shown in Figure 2. These results show that the proposed SPR-C technique is clearly more accurate than the original SPR technique. Note that the recovered stresses obtained with the proposed technique can be used to obtain accurate discretization error estimation in energy norm in this kind of problems.

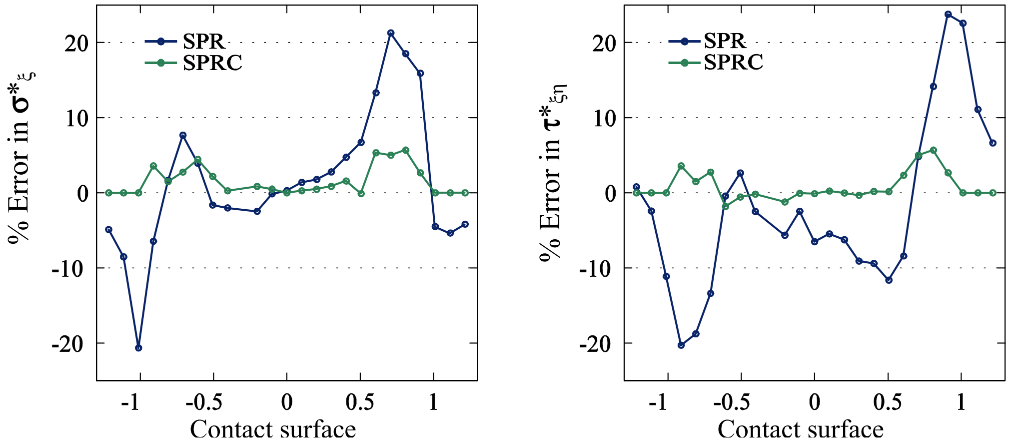

Figure 2.- Error in the recovered stresses along de contact area

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