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A *p*-VERSION FINITE ELEMENT MODEL FOR SIMPLE STRAIGHT WIRE ROPE STRANDS

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ABSTRACT

Due to increasing demand in predicting the cable behavior, several theoretical models have been built [1]. In the literature such work can not be found, which uses the *p*-version finite elements for wires strands considering the wear process.

A *p*-version of finite element model is presented for investigation of contact situation of all the possible interwire motions. The repeated slips between the core and wire cause the wear, which finally affects fretting behavior of the steel cable.

In our investigation it is proposed that the displacements and deformations are small, only the quasi static behavior of cables is addressed, the wires are made of a homogenous isotropic and linearly elastic material. For derivation of deformations the Love's curved beam equations are used.

The straight core can be considered as a particular case of a helical wire. All wires are modeled in the same way, i.e., they are considered as helical displacement-based curved beams.

In the cylindrical coordinate system (r, φ, z) the tangential $\boldsymbol{t}$, normal $\boldsymbol{n}$, and binormal $\boldsymbol{b}$ unit vectors are defined along the helix.The displacement $\boldsymbol{u}^L$ and rotation $\boldsymbol{\chi}^L$ vectors for element e may be written in the following form in above coordinate system:

$$\left[\boldsymbol{u}^T \ \boldsymbol{\chi}^T\right]^{L,T} = \boldsymbol{V}^{LG}(\varphi)\,\boldsymbol{q}_i^G + \boldsymbol{\Phi}^L(\varphi)\,\boldsymbol{a} + \hat{\boldsymbol{\Phi}}^L\,\hat{\boldsymbol{a}}\,,\ \boldsymbol{\Phi}^L(\varphi_i) = \boldsymbol{0},\ \hat{\boldsymbol{\Phi}}^L(\varphi_i) = \hat{\boldsymbol{\Phi}}^L(\varphi_j) = \boldsymbol{0}$$

where $\boldsymbol{q}_i^G$ nodal displacement vector in the first nodal point i, $\boldsymbol{V}^{LG}(\varphi)$ is the transformation matrix, which gives the rigid like displacement vector in the arbitrary point of beam, $\boldsymbol{\Phi}^L(\varphi), \hat{\boldsymbol{\Phi}}^L(\varphi)$ are the approximation matrix, $\hat{\boldsymbol{a}}$ is the vector of additional parameters, maximum number of parameters is $4 \times p = 20$, T denotes the transpose of matrices and vectors. The parameter vector $\boldsymbol{a}$ (6,1) can easily be calculated from the following condition

$$\boldsymbol{q}_j^G = \left[\boldsymbol{u}^T \ \boldsymbol{\chi}^T\right]_j^{G,T} = \boldsymbol{T}(\varphi_j)\boldsymbol{V}^{LG}(\varphi_j)\boldsymbol{q}_i^G + \boldsymbol{T}(\varphi_j)\boldsymbol{\Phi}^L(\varphi_j)\boldsymbol{a}$$

where $\boldsymbol{T}(\varphi_j)$ is the transformation matrix between the global and local coordinate system in the nodal point *j*, $\boldsymbol{q}^T = \left[\boldsymbol{q}_i^G, \boldsymbol{q}_j^G\right]^T$ is the vector of nodal displacement of the beam element. Finally in the curved beam theory three independent displacement fields u_1, u_2, u_3 and rotation χ_3 in the direction $\boldsymbol{t}$ is approximated in the form $\left[\boldsymbol{u}^T \ \chi_3\right]^{L,T} = \overline{\boldsymbol{N}}\boldsymbol{q} + \hat{\boldsymbol{N}}\,\hat{\boldsymbol{a}}$. The bending moments M_1, M_2, torsion moment M_3 and normal force F are given as $\left[F \ M_1 \ M_2 \ M_3\right]^T = \{\boldsymbol{D}[\ \overline{\boldsymbol{B}}\ \boldsymbol{q} + \hat{\boldsymbol{B}}\ \hat{\boldsymbol{a}}\ \ ]\ \}^T$, where $\boldsymbol{D} = diag(AE, IE, IE, I_pG)$ is the constitutive matrix, A is cross-sectional area, E is the Young's modulus, IE is the bending stiffness, I_pG is the torzional stiffness, $\boldsymbol{B}, \hat{\boldsymbol{B}}$ are approximation matrixes for deformations.

Using the principle of virtual displacements, taking account the Hertzian contact theory, and using the penalty technique [2] the contact stiffness matrix may be determined with iteration. Assuming the isotropic wear rule in the form $\dot{w} = \beta\,(\mu\ p_n)^b\ v_r^a = \ \tilde{\beta}\ \ p_n{}^b\ v_r^a$, where a, b, β are wear parameters, $\mu,\ p_n$ are coefficient friction and contact pressure, $\tilde{\beta} = \beta\ \mu^b$, $v_r = \left\|\dot{u}_\tau\right\|$ is the relative velocity between the bodies [3]. The shearing stress in the contact surface is calculated from the contact pressure by the Coulomb dry friction law $\tau_n = \mu\ p_n$. The wear between the core and wire can be defined by cyclic loadings after time integration, $w = \int_0^{T_*} \dot{w}\ d\tau$, where T_* is the time of observation.

The contact conditions are checked at the Lobatto integration points of the contact elements during the solution process. The oscillations of contact pressure and displacements are minimized by using the node positioning technique described in [4].

For numerical investigation a finite element program has been developed, in which the number of pitches, number of wires, number of finite elements in one pitch may be chosen arbitrarily. The boundary conditions are defined as: one end-section of the cable is fully clamped, at the other end-section the wires and core nodes are linked using rigid elements, where the master node located at the cross section center. Naturally in the master node the load can be prescribed arbitrarily by vertical force, bending and torsion moments. In the solution of contact problem the end effect gives significant influence to the distribution of contact pressure and wear rate.

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