Numerical Challenge for Option Pricing

Olivier Pironneau[†]

[†]LJLL-UPMC (Paris VI) Address Olivier.Pironneau@upmc.fr

Keywords: Scientific Computing, Finite elements, a posteriori estimate, option pricing.

ABSTRACT

For option pricing an alternative to Monte-Carlo simulation is to solve the Partial Differential Equations (PDE) derived from Ito Calculus. Finite difference methods would be sufficient normally but the power of mesh adaptivity given by the finite element method is enormous for speed and precision, especially for American options. In this talk we will introduce the method and compare it to Monte-Carlo and Finite Difference methods for vanilla Europeans with or without barriers, stochastic volatility models, models with jump processes, Asian and American options. Yves Achdou's a posteriori estimates will also be presented. Finally it will be shown how Greeks and calibration can be implemented using automatic differentiations; the importance of Dupire's equation will also be stressed for calibration and since Dupire's equation is not available for all models a discrete equivalent will be discussed.

Multidimensional problems are really challenging. They come either from complex stochatic volatilities or from multiple underlying assets. The question is weither Sparse Grid methods will ever displace Monte-Carlo methods considering the numerical and computing culture in banks which, at present, favours grid computing.

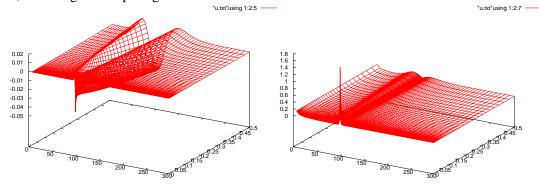


Figure 1: Error (left) compared with the a posteriori error estimate (right)

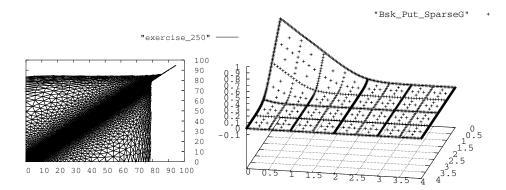


Figure 2: Left: a mesh adapted to a 2 in a basket american option (by Y. Achdou). Right: a basket option computed by the sparse grid method (by D. Pommier)

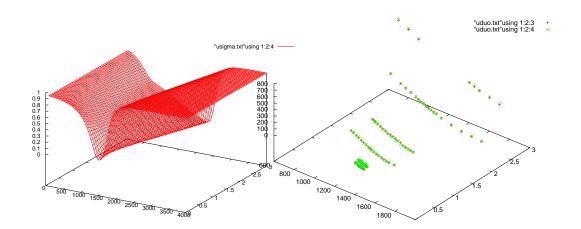


Figure 3: Volatility surface obtained by calibration (left) and difference between the data and the model predictions (right)