8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Bounds and adaptivity for 3D limit analysis

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Key Words: *adaptivity, SOCP, limit analysis, bounds, error estimates*

ABSTRACT

We briefly describe here the recent developments in the computation of bounds of the load factor λ for limit analysis and the design of adaptive remeshing strategies. Similar developments have been presented in reference [4] for 2D problems in Von Mises and Mohr-Coulomb plasticity. We extend here the essence of the method to 3D limit analysis with Von Mises plasticity. Although some three-dimensional problems can be found in the literature [2, 3, 5], none of them makes use of adaptive remeshing strategies. Since the computational cost may become in these examples exceedingly expensive after successive remeshing, the choice of an efficient error estimate is of outmost importance.

The computation of the load factor in limit analysis may be stated as the solution of the following inf-sup problem,

$$\lambda^* = \sup_{\boldsymbol{\sigma}\in\mathcal{B}} \inf_{a(\boldsymbol{\sigma},\boldsymbol{u}) = \lambda\ell(\boldsymbol{u})} \lambda = \sup_{\substack{a(\boldsymbol{\sigma},\boldsymbol{u})=\lambda\ell(\boldsymbol{u})\ \forall\boldsymbol{u}\in\mathcal{V}\\ \boldsymbol{\sigma}\in\mathcal{B}}} \lambda. \tag{1}$$

Here, $\boldsymbol{\sigma}$ and $\boldsymbol{u}$ are the stress and displacement rate field, respectively, and $a(,)$ and $\ell()$ are bilinear and linear forms, detailed for instance in [1, 4]. The set admissible stresses is defined by $\mathcal{B}$, which is here determined by the Von Mises criterium.

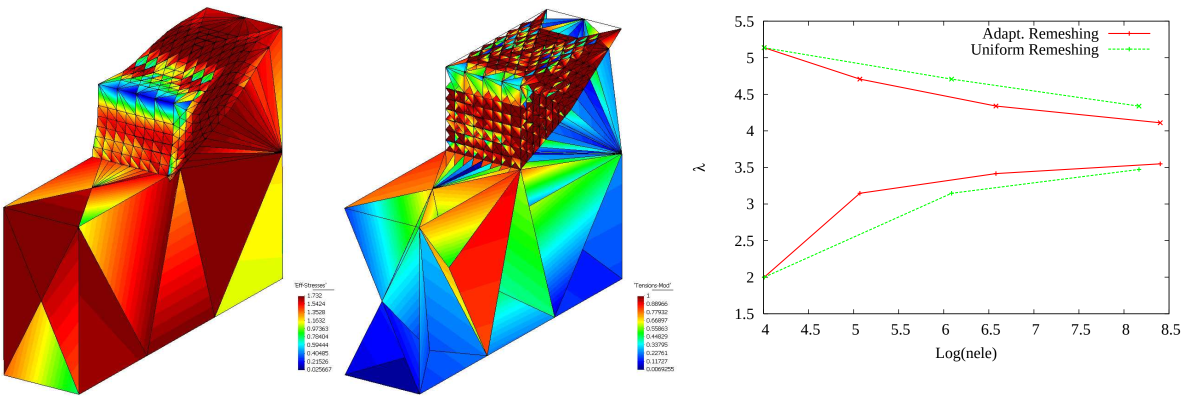
After *(i)* using a set convenient discrete spaces of the displacement rates $\boldsymbol{u}$, elemental stress $\boldsymbol{\sigma}$ and edge tension field $\boldsymbol{t}$ (similar to those used in [1, 4]), and *(ii)* introducing the transformations $\boldsymbol{\sigma} = \mathbf{T}\boldsymbol{x}$ and $\boldsymbol{t} = \mathbf{S}\boldsymbol{z}$, equation (1) turns into two min-max problems, which furnish respectively an upper bound λ^{UB} and a lower bound λ^{LB} of the load factor. Due to the transformations in *(ii)*, both problems have the form of a standard Second Order Conic Program (SOCP), i.e. the membership constraints are given by $\boldsymbol{x}^e \in \mathcal{L}^6$ and $\boldsymbol{z}^\xi \in \mathcal{L}^3$, with $\boldsymbol{x}^e$ and $\boldsymbol{z}^\xi$ the transformed elemental stresses and edge tensions, and $\mathcal{L}^n$ the n-th dimensional Lorentz cone $\mathcal{L}^n = \{\boldsymbol{x} \in \mathbb{R}^n | x_1 \geq \sqrt{\sum_{i=2}^n x_i^2}\}$.

From the primal (stresses and tensions) and dual (displacements) variables of the upper and lower bound optimisation problems, we construct elemental (Δ_λ^e) and edge contributions (Δ_λ^ξ) to the total bound gap $\Delta\lambda = \lambda^{UB} - \lambda^{LB}$. They satisfy the relations $\Delta_\lambda^e \geq 0$ and $\Delta_\lambda^\xi \geq 0$ and also

$$\Delta\lambda = \sum_{e=1}^{nele} \Delta_\lambda^e + \sum_{\xi=1}^{N_I} \Delta_\lambda^\xi.$$

Alternatively, it is possible to skip the optimisation problem for λ^{UB} and estimate the deformation rates and the load factor of the upper bound problem (the strictness of the latter is then relaxed). In this case, it is still possible to estimate the elemental and edge contributions to the bound gap, Δ_λ^e and Δ_λ^ξ, respectively. In this manner, a non-negligible reduction of the computational cost is obtained.

We have tested the formulation in the following example: a soil subjected to its own weight and with a vertical cut. This is a three-dimensional version of a similar 2D problem [2, 4], and therefore, similar slip lines should be expected. The problem has been run using an initial mesh with 55 elements. Figure 1a shows the deformed mesh after 4 remeshing process, and the values of the tangential edge tensions at the internal edges. The deformation pattern is indeed very similar to the two-dimensional case, and the numerical model is able to provide bounds of the load factor. Figure 1c shows the evolution of the upper and lower bounds. As expected, the adaptive remeshing strategy converges faster than the uniform remeshing, demonstrating the convenience for optimal strategies. Further research for other plastic criteria such as Drucker-Prager and Cam-Clay is being carried out.



(a) (b) (c)

Figure 1: Effective stresses on the deformed mesh of the upper bound problem (a) and modulus of the tangent edge tensions (b) when using 4458 elements. Evolution of the load factor when using uniform and adaptive remeshing (c).

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