

## RANS solutions and the Asymptotic Range

Eça L.<sup>1</sup> and Hoekstra M.<sup>2</sup>

<sup>1</sup> Instituto Superior Técnico, TU Lisbon  
Av. Rovisco Pais, 1, 1049-001 Lisboa  
Portugal  
eca@marine.ist.utl.pt, www.ist.utl.pt

<sup>2</sup> Maritime Research Institute Netherlands  
P.O. Box 28, Wageningen  
The Netherlands  
M.Hoekstra@marin.nl, www.marin.nl

**Key Words:** *Numerical Uncertainty, Solution Verification, RANS equations.*

### ABSTRACT

Computational Fluid Dynamics (CFD) has become an important tool in the solution of many engineering problems. Although more sophisticated methods are already available for the calculation of turbulent flows, as Direct Numerical Simulation (DNS), Large-Eddy Simulation (LES) or Detached-Eddy Simulation (DES), the Reynolds-Averaged Navier-Stokes (RANS) equations are still the most common approach in the solution of practical engineering problems.

In this paper, we will concentrate on the time-averaged RANS equations, which means that by definition the mean flow variables do not change with time. It is well known that the RANS equations do not form a closed system due to the presence of the Reynolds stresses (produced by the statistical handling of the Navier-Stokes equations). One of the most popular ways to model these unknown terms (that we will also adopt in this study) is to use the concept of eddy-viscosity, which may be computed algebraically or from the solution of one or more transport equations.

The increase of importance of CFD solutions in the development of engineering solutions requires the estimation of the numerical uncertainty of the numerical predictions. From the three components of the numerical error, round-off, iterative and discretization errors [1], the latter is usually the dominant one. Most of the techniques available for the estimation of the discretization error rely on the existence of an ‘asymptotic range’, which is basically the dominance of the lowest order term of a power series expansion of the error as a function of the typical cell size.

The existence of an ‘asymptotic range’ for steady laminar flows based on the Navier-Stokes equations has been demonstrated [1]. However, for the RANS equations (with eddy-viscosity turbulence models) its existence is hard to prove. Furthermore, doubt has been expressed on the existence of an ‘asymptotic range’ for the RANS equations [2].

The purpose of this paper is to illustrate the existence of an asymptotic range for solutions of statistically steady turbulent flows with the RANS equations complemented by eddy-viscosity turbulence models. One of the important results of this study, is that in several of the examples presented the level of grid refinement required to attain the asymptotic range is much finer than what is common practice nowadays. Therefore, the development of error estimators that perform well outside the asymptotic range is fundamental for practical calculations.

In the present study we have also tested the performance of a procedure for the estimation of the discretization error based on grid refinement studies [3] using the data of grids inside and outside the 'asymptotic range'.

## REFERENCES

- [1] Roache P.J. - Verification and Validation in Computational Science and Engineering - Hermosa Publishers, 1998.
- [2] Hirsch C. - Private communication.
- [3] Eça L., Hoekstra M. - Discretization Uncertainty Estimation based on a Least Squares version of the Grid Convergence Index - 2<sup>nd</sup> Workshop on CFD Uncertainty Analysis, Instituto Superior Técnico, Lisbon, October 2006.