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A GENERALIZED DIFFERENTIAL QUADRATURE SOLUTION FOR LAMINATED COMPOSITE SHELLS OF REVOLUTION

Francesco Tornabene[1], Erasmo Viola[2]

[1] DISTART - Department, University of Bologna, Italy, v.le Risorgimento 2, 40136 Bologna
francesco.tornabene@mail.ing.unibo.it

[2] DISTART - Department, University of Bologna, Italy, v.le Risorgimento 2, 40136 Bologna
erasmo.viola@mail.ing.unibo.it

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ABSTRACT

In this paper, the Generalized Differential Quadrature (GDQ) Method [1] is applied to study laminated composite shells of revolution. The mechanical model is based on the so called First-order Shear Deformation Theory (FSDT), deduced from the three-dimensional theory, in order to analyze the above moderately thick structural elements.

The governing equations of motion are expressed as functions of five kinematic parameters, by using the constitutive and the kinematic relationships. The solution is given in terms of generalized displacement components of points lying on the middle surface of the shell.

As it is well known, the GDQ method was developed to improve the Differential Quadrature (DQ) method for the computation of weighting coefficients [1]. By using the GDQ technique the numerical statement of the problem does not pass through any variational formulation, but deals directly with the governing equations of motion [2-6]. Referring to the formulation of the equilibrium equations of doubly curved shells, in this study the system of second-order linear partial differential equations is solved. The discretization of the system by means of the GDQ procedure leads to a linear system of equations, where two independent variables are involved.

The results are obtained taking the meridional and the circumferential co-ordinates into account, without using the Fourier modal expansion methodology [7]. Complete revolution shells are obtained as special cases of shell panels by satisfying the kinematical and physical compatibility at the common meridian characterized by $\vartheta = 0, 2\pi$.

After the solution of the fundamental system of equations in terms of displacements, the generalized strains and stress resultants can be evaluated by applying the Differential Quadrature rule [1] to the displacements themselves. However, in order to design composite laminate structures properly, accurate stress analyses have to be performed.

The significant shear deformation in the thickness and extensional-bending coupling should be considered. In fact, the determination of accurate values for interlaminar normal and shear stresses is of crucial importance, since they are responsible for the activation and the development of delamination mechanisms. In this work, the

transverse shear and normal stress profiles through the laminate thickness are reconstructed a posteriori by simply using local three-dimensional equilibrium. No preliminary recovery or regularization procedure [8-10] on the extensional and flexural strain fields is needed when the Differential Quadrature technique is used. An accurate estimation of derivatives of the quantities involved in solving the three-dimensional equilibrium equations is obtained. By using GDQ procedure through the thickness, the reconstruction procedure needs only to be corrected to properly account for the boundary equilibrium conditions and static equivalence with shear forces.

In order to verify the accuracy of the present method, GDQ results are compared with the ones obtained using analytical solutions for composite laminate plates [11]. Very good agreement is observed. Examples of shear stress profile for plate elements are presented to illustrate the validity and the accuracy of GDQ method.

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