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A TWO-SCALE APPROACH TO CONTACT PROBLEMS BETWEEN A RIGID ROUGH SURFACE AND AN ELASTIC OR VISCOELASTIC HALF-SPACE

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ABSTRACT

This paper presents a new numerical approach to contact problems between a rough surface of a rigid body and a smooth surface of an elastic or viscoelastic half-space. The number of asperities on the rough surface may be large and classical numerical methods are not efficient for computing the contact pressure distribution. A large literature base is available for a variety of numerical algorithms and can be found for example in the monograph of Wriggers [1].

The Two-scale approach proposed here proceeds successively at two scales, the first one called the macro-scale and the second the micro-scale. During the macro-scale step, the forces on each asperity are computed from an approximate relation between the normal contact force and the relative displacement at the tip of each asperity. The approximate relation takes into account the interacting effect between the asperities. In fact, when an elastic medium is pressed on the rigid asperities with a global penetration δ, the normal contact force P_k on the k^{th} asperity can be approximatively calculated as follows:

$$P_k = f_k\left[(\delta - z_k - u_k)H(\delta - z_k - u_k)\right] \tag{1}$$

where H is the Heaviside's function defined by $H(y) = 0$ if $y < 0$ and $H(y) = 1$ if $y \geq 0$, u_k represents the normal displacement of the elastic half-space at the tip of the k^{th} asperity due to the other asperities and f_k is the load-penetration relationship for the k^{th} asperity alone. This relationship can be an analytical function or a numerically simulated function. According to the superposition principle in the linear elasticity theory, this displacement is the algebraic sum of the displacement due to the force on each asperity:

$$u_k = \sum_{l=1}^{N} T_{kl}P_l = T_{kl}P_l \quad \text{with } T_{kk} = 0 \tag{2}$$

where N is the number of asperities, T_{kl} is an influence constant depending on the distance r_{kl} between the k^{th} asperity tip and the l^{th} asperity tip projected on the plane surface of the half-space. Assuming that the local contact areas are small, the influence coefficients T_{kl} can be estimated using the solution of the Boussinesq's problem for a concentrated normal load applied on an elastic half-space. Combining these two equations leads to a system of N nonlinear equations with N unknowns P_1, P_2, ... , P_N:

$$\forall k \in [1, N], \qquad P_k - f_k \left[(\delta - z_k - T_{kl} P_l) H(\delta - z_k - T_{kl} P_l) \right] = 0 \tag{3}$$

Such a system of nonlinear equations can be solved using the Newton-Raphson iterative method. In this way, each normal contact force P_k can be related to the value of the global penetration δ. When the total force P is given, δ can be considered as an unknown. Since the overall equilibrium requires that the sum of P_k equals the total force, a system of $N + 1$ equations with $N + 1$ unknowns P_1, P_2, ... , P_N and δ can be solved.

Then at the micro-scale step, one can construct an iterative algorithm by using a numerical method for computing the pressure distribution on each asperity alone. The pressure obtained at the precedent iteration on other asperities are used to compute their influence on the asperity under consideration. At the first iteration, the contact forces computed at the macro-scale step are used to compute this influence. So the efficiency of the Two-scale approach rely on the approximation quality at the macro-scale step.

The Two-scale approach has been applied to contact problems between a rigid rough surface composed of periodically or randomly distributed spherical asperities. The results agrees well with those of others numerical methods but our algorithm is much more rapid. The algorithm has also been used to calculate the pressure between a smooth tyre and pavements. The numerical results agrees well with that of experimental measurements.

We are now studying the extension of the Two-scale approach to viscoelastic contact problems such as the contact between tyres and pavements. The contact problem between a rigid body and a viscoelastic half-space is governed by the following equation

$$u(x, y, t) = \frac{(1-\nu)}{\pi} \int_0^t J(t-\tau) \left[\iint_{\Omega_m} \frac{1}{\rho} \frac{dp(\xi, \eta, \tau)}{d\tau} d\xi d\eta \right] d\tau \tag{4}$$

where the displacement $u(x, y, t)$ must satisfy the contact boundary conditions, $J(t)$ is the creep function, ρ is the distance between the points (x, y) and (ξ, η), $p(x, y, t)$ is the contact pressure distribution history and Ω_m is the maximum contact area. If the pressure history is known from 0 to t and the pressure distribution is to be computed from t to $t + \Delta t$, one can divide the time integral into two parts: one from 0 to t and another from t to $t + \Delta t$. By supposing that the changing rate of the pressure is constant at any point for a small time increment Δt, the problem of determining this changing rate becomes similar to an elastic problem of determining the pressure distribution. Then a similar Two-scale approach can be applied. In this way, the contact problem between a rigid rough surface and a viscoelastic half-space can be solved.

REFERENCES

[1] P. Wriggers, *Computational Contact Mechanics*, 2nd Edition, Springer, 2006.