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A stabilized scheme for the Primitive Equations of the Ocean based upon orthogonal subscales

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ABSTRACT

The Primitive Equations of the ocean are approximations of the Navier-Stokes equations based upon two hypotheses: Hydrostatic pressure and rigid-lid. These equations are extensively used in Physical Oceanography and numerical simulation of sea flow as a model for large-scale (in time and space) flows, whenever the thickness of the flow is small compared to its horizontal dimensions. An asymptotic obtention of the Primitive Equations from the Navier-Stokes equations as the ratio vertical diameter of domain/horizontal diameter of domain tends to zero is performed in Besson and Laydi [1].

In mathematical terms, the Primitive Equations may be formulated as a mixed problem whose unknowns are the horizontal velocity and the ”surface pressure”. This is a virtual pressure that must be applied on the surface to keep it flat. The vertical velocity is recovered from the incompressibility restriction. The equations are more singular than the Navier-Stokes equations (the horizontal velocity has H^1 regularity, while the vertical velocity is only in L^2 with vertical derivative in L^2), needing a Petrov-Galerkin formulation where the test functions are more regular than the solution. For this reason, the inf-sup condition yielding the stability of the pressure holds in L^p norm, for some $p < 2$.

This lack of regularity makes the numerical approximation of the Primitive Equations more difficult than that of Navier-Stokes equations, although the number of unknowns required is about one half of that required by Navier-Stokes equations. A further reduction of the number of degrees of freedom is obtained with respect to mixed methods if stabilized methods are used. An analysis of such a kind of approximations may be found in Chacón and Rodríguez [3], where an approximation by prismatic finite elements is developed. In that paper, standard values for the stabilization coefficients are used, and a large sensibility of the numerical solution with respect to these coefficients is found.

This talk is aimed to present an improved stabilized method, where the stabilization coefficients are in some sense optimized. We adapt the deduction of these coefficients performed by Codina in [4], in a context of orthogonal sub-scales. We perform a convergence analysis by the technique developed in Chacón [2], where stabilized methods are characterized as mixed methods for a specific ”augmented”

variational formulation, intrinsically stable. We finally present some numerical tests for meaningful flows that show the improvements achieved with the optimized stabilization coefficients.

REFERENCES

[1] O. Besson, M.R. Laydi, “Some estimates for the anisotropic Navier-Stokes Equations and for the hydrostatic approximation”, *Math. Mod. and Num. Anal.* Vol. **26**, 7, 855-865. 1992.

[2] T. Chacón Rebollo, “An analysis technique for Stabilized Finite Element solution of Incompressible Flows”. *M2AN*, **Vol. 35**, 57-89, 2001.

[4] T. Chacón Rebollo, D. Rodríguez Gómez, “A numerical solver for the Primitive Equations of the Ocean using term-by-term stabilization”, *Appl. Num. Math.*, Vol. **55** No. 1, 1-31, 2005.

[5] R. Codina.“Stabilized finite element approximation of transient incompressible flow using orthogonal subscales”, *Comput. Methods Appl. Mech. Engrg.*, Vol. **191**, 4295-4321, 2002.