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A SYMMETRIC FORMULATION FOR NONLOCAL ELASTIC FINITE ELEMENT ANALYSIS

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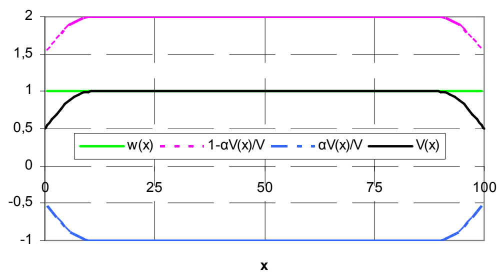
ABSTRACT

It is well-known that one of the main drawbacks of local elasticity consists in the fact that many problems, such as sharp crack-tip in continuum fracture mechanics, lead to stress singularities in classical elastic theories. A possible solution consists in considering a continuum approach in which there are information regarding the behaviour of the material microstructure by assuming that an elastic material can transmit information to neighbouring points within a certain distance. Such a distance is the internal length scale and is an essential material parameter which accounts for nonlocal effects in the continuum. Nonlocal variables turn then out to be weighted average of the corresponding local variables over the material points of the structure and the internal length controls the weighting process related to a state variable at a given point. A continuum theory for elastic material with long range cohesive forces can be found in the pioneristic work of Kröner [1]. A nonlocal elastic theory is presented in [2] but a simplified and more effective nonlocal theory is contributed in [3] by assuming that nonlocality appears only in the constitutive relation. It is shown that several problems related to stress singularities in local elasticity, such as crack-tip problems, disappear by adopting the nonlocal theory, see e.g. [4]. An elastic model in a geometrically linear range endowed with the nonlocal elastic material model proposed in [3,5] is dealt with in [6] in which the extension to nonlocal linear elasticity of the classical principles of the total potential energy, complementary energy and mixed Hu-Washizu principle are also provided. Further developments on the subject of nonlocal elasticity can be found in [7]. In the present paper, starting from the nonlocal elastic constitutive model proposed by Eringen and co-workers, the thermodynamic framework and the boundary-value problem for nonlocal elasticity are formulated and the complete set of nonlocal mixed variational principles is then provided. A recently proposed, in the context of damage mechanics [8], symmetric spatial weight function which preserves constant fields is considered. A firm variational basis to the nonlocal model is provided by showing that the admissible combinations of the state variables yield six nonlocal mixed variational formulations and four nonlocal one-field variational principles. A discussion on uniqueness of the solution of the nonlocal model is then provided. The variational formulations for nonlocal linear elasticity presented in [6], following an ad hoc reasoning, are then recovered. A consistent symmetric nonlocal FE procedure is then derived. A piecewise homogeneous bar is solved by the recourse to the Fredholm

equation and to the proposed nonlocal FEM for several load conditions. The solutions obtained following the outlined procedures are in a good agreement each other and no pathological behaviours at the boundary are present.



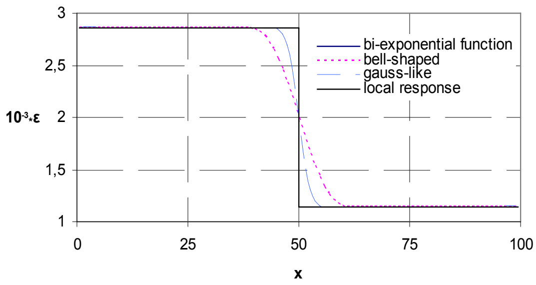


Fig. 1a

Fig. 1b

The data used for the computation of the one-dimensional bar of length L=100 cm are: elastic modulus $E_o=21\times10^4$ MPa, internal length l=2 cm, influence distance R=12 cm and material parameter α=-1. The imposed displacement at the end $x=L$ is given by w=0.2 cm. In Fig. 1a it is reported the plot of $w(x)$ which is the integral on the bar of the symmetric weight function $W(x,y)=(1-\alpha V(x)/V)\delta(x,y) + \alpha/Vg(x,y)$ with respect to the variable y and the plots of the two contributions $(1-\alpha V(x)/V)$ and $\alpha V(x)/V$ and of the representative volume $V(x)$ for the bi-exponential attenuation function g. In Fig. 1b the strain plots of the bar in tension for different attenuation functions g in the expression of the symmetric function W and for a piecewise homogeneous bar with $E=0.4E_o$ for x belonging to $[0,L/2]$ and $E=E_o$ for x belonging to $[L/2,L]$ are reported.

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