8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

On the solution of Navier-Stokes equations by the PSI method as a non-linear Petrov-Galerkin method

Marta Benítez García[1],* Tomás Chacón Rebollo[2], Macarena Gómez Mármol[2] and Gladys Narbona Reina[3]

[1] Dpto. de Matemática Aplicada
Universidad de Santiago de Compostela
Facultad de Matemáticas. Rùa Lope Gómez de Marzoa, s/n. Campus sur 15782 Santiago. (Spain)
martabg@usc.es

2 Dpto. de Ecuaciones Diferenciales y Análisis Numérico
Universidad de Sevilla
Facultad de Matemáticas. C/ Tarfia, s/n. 41012 Sevilla (Spain)
chacon@us.es, macarena@us.es

[3] Dpto. de Matemática Aplicada I
Universidad de Sevilla
Escuela Superior de Arquitectura. 41012 Sevilla (Spain)
gnarbona@us.es

Key Words: *Residual distribution methods, Petrov-Galerkin formulation, Navier-Stokes equations, Method of Characteristics*

ABSTRACT

The numerical solution of Navier-Stokes equations shares with that of advection-diffusion equations the challenge of designing methods that combine high accuracy at steady state with stability for convection-dominant flows. Due to Godunov's theorem, non-linear methods are needed to comply with both requirements. Among these methods we may cite that of Characteristics (Cf. Pironneau [5]), and more recently Discontinuous Galerkin (Cf. Cockburn [3]) and Fluctuation Splitting (FS) methods (Cf. Deconninck et al. [4]).

This work is aimed to analyze the solution of Navier-Stokes equations by the PSI (Positive Streamwise Implicit) and some other non-linear Fluctuation Splitting methods. FS methods apply to finite element space discretizations. Their main design idea is to split the element residual on the nodes of the element that are downwind. The PSI method is specifically designed to be exact for linear solutions of the pure transport problem. It is monotone and second order accurate for steady solutions of advection-diffusion equations. Its advantage with respect to the method of Characteristics, for instance, is to provide a more accurate and monotone solution of zones of the flow with sharp gradients. However, it is somewhat more diffusive than second-order characteristics.

Our analysis is based upon a re-formulation of nonlinear FS methods as Petrov-Galerkin methods. This allows the use of standard tools of functional analysis to perform its numerical analysis. In Chacón et al. [2] an analysis of the solution of advection-diffusion equations is performed by this technique. This analysis is based upon the construction of PSI method from the first-order N-scheme due to Abgral et al. [1]. We extend here this analysis to steady Navier-Stokes equations. We prove convergence and a priori error estimates for piecewise affine finite element discretizations of the velocity. We prove that

it is well balanced up to second order. We also present some numerical tests in meaningful flows that confirm the theoretical predictions and compare it with the method of Characteristics.

REFERENCES

[1] R. Abgrall and M. Mezine. “Construction of second-order accurate monotone and stable residual distribution schemes for unsteady flow problems”. *J. Comput. Phys.*, Vol. **188**, 16-55, 2003.

[2] T. Chacón Rebollo, M. Gómez Mármol, G. Narbona Reyna. “Numerical analysis of the PSI solution of Advection-Diffusion problems through a Petrov-Galerkin formulation”,*M3AS*, Vol. **17**, No. 11, 1905-1936. 2007.

[3] B. Cockburn. “An introduction to the Discontinuous Galerkin method for advection-dominated problems”, in *Advanced Numerical Approximation of Nonlinear Hyperbolic Equations (Cetaro 1997)*. Lecture Notes in Math., Vol. **1697**, 1998.

[4] H. Deconinck, H. Paill‘ere, R. Struijs and P. L. Roe.“Multidimensional upwind schemes based upon fluctuation-splitting for systems of conservation laws”, *Comput. Mech.*, Vol. **11**, 323-340, 1993.

[5] O. Pironneau.“On the transport-diffusion algorithm and its application to the Navier-Stokes equations”, *Numer. Math.*, Vol. **38**, 309-332, 1981/82.