8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Numerical analysis of a bone remodeling problem

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Key Words: *Bone remodeling, nonlinear adaptive elasticity, fully discrete approximations, error estimates, numerical simulations.*

ABSTRACT

In this talk, we study from the numerical point of view the evolution of the remodeling of a bone which is governed by a nonlinear adaptive elasticity model. According to [1], this bone remodeling model is is a generalization of the nonlinear elasticity, and it is based on the fact that the living bone is continuously adapting itself to external stimuli. Since this process has an enormous effect on the overall behavior and health of the entire body, the ability of these models to predict the bone remodeling is of great importance.

During the last ten years, several papers dealt with mathematical issues of these models as the existence and uniqueness of weak solutions under some quite strong assumptions (see, e.g., [4] or [5]) or the analysis of an asymptotic one-dimensional problem ([2,3]). In this work, we consider the mechanical problem, and its associated variational formulation, in the d-dimensional setting, and we provide its numerical analysis and we perform some numerical simulations in one, two and three dimensions.

The variational problem is written as a linear variational equation for the displacement field, coupled with a first-order ordinary differential equation to describe the physiological process of bone remodeling. An existence and uniqueness result of weak solutions was stated in [4] by using Schauder's fixed point theorem, the Cauchy-Lipschitz-Picard theorem, the Lax-Milgram lemma and regularity results.

Here, we introduce a fully discrete approximation based on the finite element method to approximate the spatial variable and an Euler scheme to discretize the time derivatives. A main error estimates result is obtained, from which the linear convergence of the algorithm is derived under suitable regularity conditions on the continuous solution. Finally, some numerical results, in one, two and three dimensions, are presented in order to demonstrate the performance and the accuracy of the algorithm. As an example, in Figure 1 we provide the results obtained in a three-dimensional example, for a compression force linearly increasing through the vertical direction but time-independent, and applied on the upper horizontal boundary. As can be seen, the displacements decrease due to the bone remodeling.

Figure 1: Reference configuration and displacement field at initial time (left) and after 66 days (right).

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