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Numerical analysis of a dynamic viscoelastic contact problem with damage

Marco Campo, José R. Fernández and Juan M. Viaño

Departamento de Matemática Aplicada, Universidade de Santiago de Compostela
Facultade de Matemáticas, Campus sur s/n, 15782 Santiago de Compostela, Spain
E-mail: macampo@usc.es, jramon@usc.es, maviano@usc.es

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ABSTRACT

In this work we numerically study a model for the process of unilateral frictionless contact between a viscoelastic body and a deformable foundation. Material damage due to the opening and growth of micro-cracks and micro-cavities caused by compression or tension is taken into account. Reliable prediction of the development of material damage in engineering systems which undergo cyclic loadings is of considerable importance, and the effective functioning, reliability and safety of the system may be affected by the decrease in its load carrying capability, as the material damage evolves. Because of the importance of the subject, an increasing number of mathematical and engineering publications deal with it (see, e.g., [3] and the references cited therein).

The variational problem is written in the following form (see [1,4] for details concerning the notation and the operators).

Problem VP. *Find a velocity field* $\boldsymbol{v} : [0,T] \to V$ *and a damage field* $\zeta : [0,T] \to \mathcal{K}$ *such that* $\boldsymbol{v}(0) = \boldsymbol{v}_0$, $\zeta(0) = \zeta_0$ *and for a.e.* $t \in [0,T]$,

$$\begin{aligned}
&\langle \rho\dot{\boldsymbol{v}}, \boldsymbol{w}\rangle_{V'\times V} + (\zeta(t)\mathcal{A}(\boldsymbol{\varepsilon}(\boldsymbol{v}(t))), \boldsymbol{\varepsilon}(\boldsymbol{w}))_{[L^2(\Omega)]^{d\times d}} \\
&\qquad = \langle \boldsymbol{f}(t), \boldsymbol{w}\rangle_{V'\times V} - j(\zeta(t); \boldsymbol{u}(t), \boldsymbol{w}) - (\zeta(t)\mathcal{G}(\boldsymbol{\varepsilon}(\boldsymbol{u}(t))), \boldsymbol{\varepsilon}(\boldsymbol{w}))_{[L^2(\Omega)]^{d\times d}} \quad \forall \boldsymbol{w} \in V, \\
&(\dot{\zeta}(t), \xi - \zeta(t))_{L^2(\Omega)} + a(\zeta(t), \xi - \zeta(t)) \geq (\phi(\boldsymbol{\varepsilon}(\boldsymbol{u}(t)), \zeta(t)), \xi - \zeta(t))_{L^2(\Omega)} \quad \forall \xi \in \mathcal{K},
\end{aligned}$$

where $\mathcal{A}$ and $\mathcal{G}$ denote the viscosity and elasticity fourth order tensors, respectively, employed to define the classical Kelvin-Voigt material law (see [2]), the displacement field is defined as

$$\boldsymbol{u}(t) = \int_0^t \boldsymbol{v}(s)ds + \boldsymbol{u}_0,$$

the normal compliance operator $j : \mathcal{K} \times V \times V$ has the form (p is the normal compliance function defined in [5]),

$$j(\xi; \boldsymbol{w}, \boldsymbol{v}) = \int_{\Gamma_C} \xi p(w_\nu - g)v_\nu \, da \quad \forall \boldsymbol{w}, \boldsymbol{v} \in V, \forall \xi \in \mathcal{K},$$

the variational space V and the convex set $\mathcal{K}$ are given by

$$
\begin{aligned}
V &= \{\boldsymbol{v} \in [H^1(\Omega)]^d \,;\ \boldsymbol{v} = \boldsymbol{0} \quad \text{on} \ \ \Gamma_D\}, \\
\mathcal{K} &= \{\xi \in H^1(\Omega)\,;\, \zeta_* \leq \xi \leq 1 \quad \text{a.e. in} \quad \Omega\},
\end{aligned}
$$

the bilinear form $a : \mathcal{K} \times \mathcal{K} \to \mathbb{R}$ is defined by

$$
a(\xi, \eta) = \kappa \int_\Omega \nabla\xi \cdot \nabla\eta \, d\boldsymbol{x} \quad \forall \xi, \eta \in \mathcal{K},
$$

$\kappa > 0$ represents the damage difussion constant and ϕ denotes the damage source function.

The existence of a unique weak solution to Problem VP was proved in [4]. In this talk, our aim is to introduce a fully discrete scheme for solving this variational problem based on the finite element method to approximate the spatial variable and the Euler scheme to discretize the time derivatives. Then, error estimates on the approximate solutions are provided from which, under some additional regularity assumptions, the linear convergence of the algorithm is deduced. Finally, some numerical simulations are shown to demonstrate the performance of the method. As an example, we consider a viscoelastic wrench and so, in Figure 1 we plot the von Mises norm for the stresses (left) and the damage field (right), on the deformed mesh, at final time. As we expected, the highest stresses are concentrated near the contact part (around the screw) and where the wrench bends, and these areas coincide with the the more damaged ones.

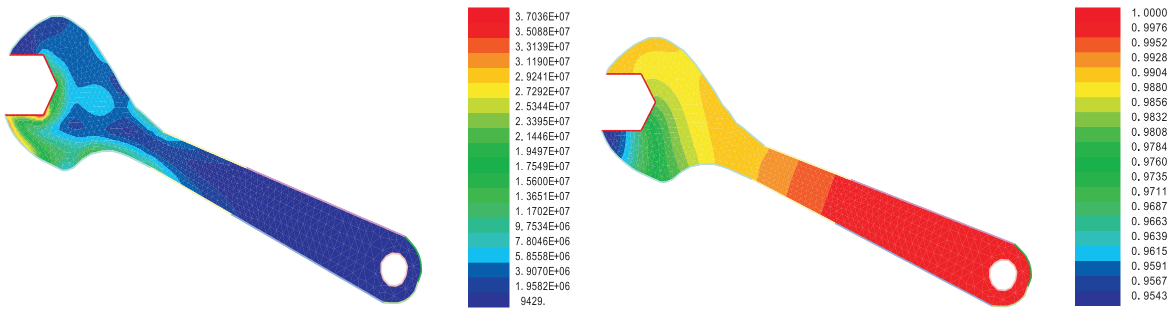

Figure 1: Von Mises stress norm (left) and damage field (right) on the deformed mesh at final time.

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