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RESIDUAL BASED FORMULATIONS OF SPACE-TIME DISCONTINUOUS GALERKIN METHODS FOR ELASTO-DYNAMIC PROBLEMS

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ABSTRACT

The numerical approximation of the dynamic deformation of a solid has been the subject of an extremely vast literature.

By far, the most widely used approximation scheme is based on semi-discretization, whereby finite elements are used in space and finite differences are used in time. While conceptually very simple, this approach eliminates the ability to produce local mesh refinement in the space-time domain, which is one of the main reasons to use a combined space-time approximation.

While there are several papers on space-time approximations of first order hyperbolic equations (see [9] for a list of references), the reduction of the elasto-dynamic problem to a system of first order equations, harshen one of the main motivation for the avoidance of the full space-time finite element discretization, namely the strong computational restrictions on computer memory usage.

A first answer in this direction comes from the early paper by Hughes and Hulbert [8], where second order hyperbolic-problems are treated directly and the computational domain is sliced in time-slabs which are solved sequentially, exploiting the hyperbolic nature of the problem.

This approach gives rise to a method which is discontinuous in time, and where the solution at the end of each time-slab is used as initial data for the following slab. The analysis in [8] is aided by the addition of carefully crafted least square terms that produce some limitations on the characteristic of the space-time mesh, but allows for a rigorous error estimate.

On a similar track we find the works by French *et al.* [6, 5] where the limitations on the space-time mesh are eliminated by a modification of the variational formulation and by the use of continuous space-time finite elements.

In [7] and [3] a generalization of [8] was proposed which exploits more deeply the superiority of discontinuous Galerkin methods for hyperbolic problems, allowing the material to undertake solid-solid phase changes. In particular the stabilization terms added in [7, 3] are not of the least square type, but are based on physically related quantities such as the energy release rate. The same ideas were extended to thermo-elasto-dynamic problems in [4].

In this work we present a class of residual based formulations for second order hyperbolic problems based on a space-time adaptation of the concepts found in Brezzi *et al.* [2], which includes as special cases all the aforementioned methods.

In particular we select a novel method which is discontinuous both in space and time and we apply it to solid-solid phase changes as well as to simple dynamic crack propagation problems.

We provide several numerical examples, and we set the stage for a detailed convergence and a posteriori analysis of these methods, following some ideas found in [1] and analyzing the choice of a posteriori error estimates presented in [4].

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