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Intrusive Stochastic Finite Element Methods - Computational Aspects and Applications

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ABSTRACT

The robustness of finite element (FE) model-based predictions with respect to the uncertainties associated with the problem, can be studied by modelling the parameters of the FE model as random variables and random processes. This leads to a family of methods referred to as stochastic finite element methods (SFEM). The various SFE methods differ mainly in the way the stochastic response quantities of interest are represented and computed.
Besides the Monte Carlo simulation-based methods, which can be applied most generally, several expansion-based methods have been developed, such as the perturbation method [1], the Neumann expansion [2] and the Spectral SFEM [3], in which the stochastic response is represented with respect to a particular orthogonal basis. In general, expansion-based SFEM rely on expressing the system matrices (e.g. stiffness matrix) of FE model in terms of expansions,

$$\mathbf{K}(\theta) = \sum_{i=0}^{l} \xi_i(\theta)\mathbf{K}_i \, , \tag{1}$$

and lead to a general expression for the response of the form

$$\mathbf{u}(\theta) = \sum_{j=0}^{p} \eta_j(\theta)\mathbf{u}_j \, , \tag{2}$$

where θ denotes the randomness; $\xi_i(\theta)$, $\eta_j(\theta)$ are random coefficients and $\mathbf{u}_j$, $\mathbf{K}_i$ are deterministic vectors and matrices respectively. The vectors $\mathbf{u}_j$ define the stochastic response.
From the software implementation point of view, SFE methods can be classified in two distinct categories, intrusive methods and non-intrusive methods. Intrusive methods require access to the system matrices whereas for the non-intrusive methods, the FE solver can be treated as a ”Black-Box”. This representation focuses on the implementation of the aforementioned intrusive SFE methods in commercial FE solvers, a factor which is of critical importance for the penetration of SFEM in the community

of industrial users.
The approach taken in this case differs from previous ones [4,5], in that the stochastic response representation and evaluation is pursued within the commercial FE code. This results in an improved efficiency thanks to the reduced data traffic from and to the FE solver (Figure 1).
The efficiency of the alternative approaches to the implementation of intrusive SFEM is investigated in context with aerospace applications of industrial relevance. It can be concluded that due to the reduced export and import of system matrices, significant gains in the efficiency can be accomplished.

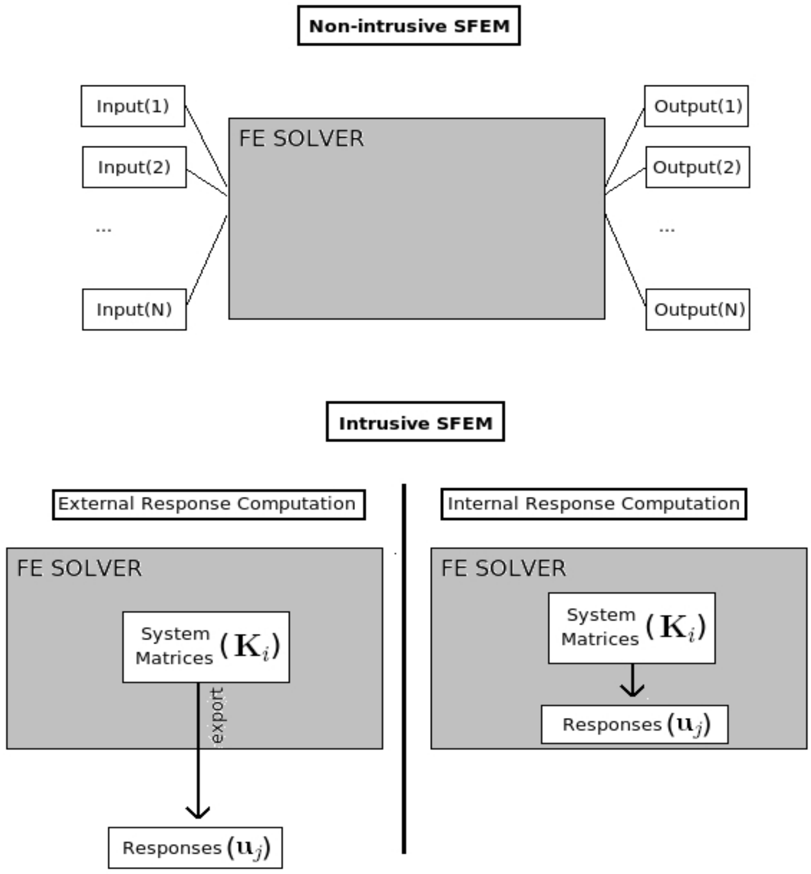

Figure 1: Schematic representation of intrusive and non-intrusive SFEM

REFERENCES

[1] M. Kleiber, T.D. Hien. *The stochastic finite element method*, Wiley, 1992.

[2] F. Yamazaki, M. Shinozuka, G. Dasgupta. “Neumann expansion for stochastic finite element analysis”. *Journal of Engineering Mechanics*, ASCE 114 (8), 1335-1354, 1988.

[3] R. Ghanem, P. Spanos. *Stochastic finite elements: a spectral approach*, New York, Springer, 1991.

[4] G.I. Schuëller, H.J. Pradlwarter. “Computational stochastic structural analysis (COSSAN) - a software tool”. *Structural Safety 28*, 68-82, 2006.

[5] S.K. Sachdeva, P.B. Nair, A.J. Keane “On using deterministic FEA software to solve problems in stochastic structural mechanics”. *Computers & Structures, 85*, 277-290, 2007.