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Random eigenvalue problem in the robustness analysis of large-scale structures

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ABSTRACT

Model-based predictions of structural behavior are negatively affected by uncertainties of various type and in various stages of the structural analysis. The present paper focusses on dynamic analysis and addresses the robustness of the predictions with respect to uncertainties in the material and geometrical parameters, mainly in the context of modal analysis of large-scale structures from aerospace engineering. Given the large number of uncertain parameters arising in this case, highly scalable simulation-based methods are adopted, whose performance does not deteriorate under these circumstances.

Monte-Carlo simulation (MCS) procedures to assess uncertainty propagation consist in the generation of sample populations that are consistent with the probability distribution of the input data (loading) and/or of the system properties (structural parameters). From a practical and computational point of view two major assets of the method are the following: (i) the non-intrusive nature of MCS facilitates its use in a black-box fashion, in combination with general-purpose finite element codes [1]; (ii) the MCS algorithm is, due to the generation of independent samples, particularly well suited for parallel processing and can hence take full advantage of high-performance computing facilities that are becoming increasingly affordable and widespread.

Modal analysis, i.e. the act of determining the eigenfrequencies of structures and the associated modes of deformation, is of fundamental importance in the construction of predictive FE-models of structures subjected to dynamic excitation. For a given FE model, modal analysis consists in solving the generalized eigenvalue problem associated with the linear elasto-dynamic equation of motion,

$$(-\omega_j^2 \mathbf{M} + \mathbf{K})\phi_j = 0\,. \tag{1}$$

where $\{\boldsymbol{\omega}_j\}_{j=1}^N$ and $\{\boldsymbol{\phi}_j\}_{j=1}^N$ are the sets of eigenfrequencies and normal modes, respectively, resulting from the solution of the above eigenvalue problem.

The validation of such FE-models is typically based on the correlation of experimental and numerical results. More specifically, the modal properties predicted by the FE model and those emerged in experimental testing campaigns are correlated (see e.g. [2]).

The representation of the uncertainties in the structural parameters in a probabilistic framework, results obviously in the propagation of these uncertainties (according to eqn. (1)) to the eigenfrequencies and eigenmodes. In the context of simulation methods, this uncertainty propagation - albeit conceptually simple - poses significant challenges if applied to large-scale structures: i) the simulation has to be performed in an efficient to avoid excessive computational cost; ii) for closely-spaced eigenfrequencies, it is necessary to account for the mode-intermixing. The solution here adopted represents each eigenfrequency as a weighted superposition of the remaining eigenfrequencies.

To demonstrate the applicability of the adopted approaches to large-scale problems of industrial interest, the effects of uncertainties on complex structural systems are analyzed in the context of the INTEGRAL satellite of ESA (European Space Agency), shown in the left portion of Fig. 1. In total, the FE model of the satellite involves 120,000 DOFs. Since the adopted probabilistic modelling process has been conservative in the sense that the uncertainty in as many parameters as practically possible has been accounted for, a very large number of random variables has been introduced in the model, namely more than 1,300.

The direct Monte-Carlo simulation of the eigenfrequencies of the satellite structure provides the reference solution. The corresponding results for the first four eigenfrequencies are summarized by the histograms in the right portion of Fig. 1.

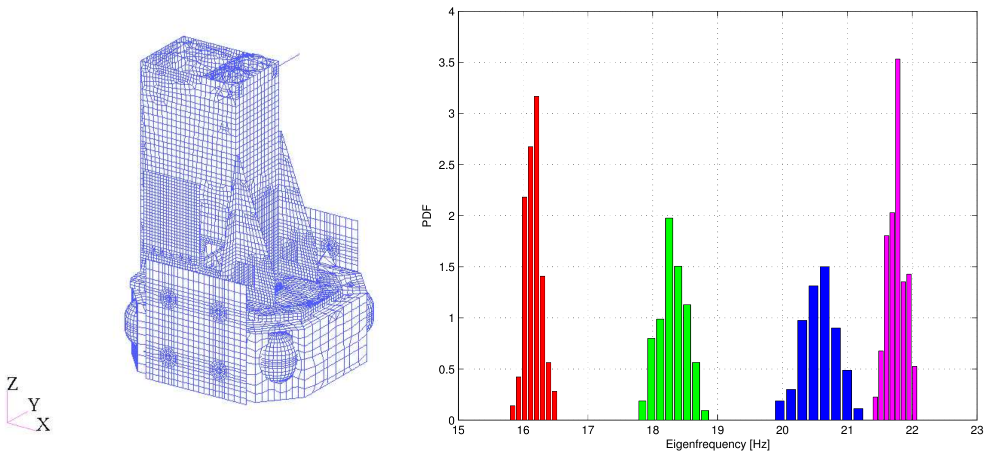

Figure 1: Left: Satellite finite element model (courtesy of ESA/ESTEC); right: Histograms of eigenfrequencies, modes 1-4, based on Direct MCS (200 samples)

References

[1] Pellissetti, M. F. and Schuëller, G. I., "On general purpose software in structural reliability," *Structural Safety*, Vol. 28, No. 1–2, 2006, pp. 3–16.

[2] Ewins, D. J., *Modal testing : theory, practice, and application*, Research Studies Press, Baldock, 2nd ed., 2000.