

## **A non-iterative FEM-based cavity identification method using topological sensitivity for 2-D and 3-D time-domain elastodynamics**

**C. Bellis and M. Bonnet**

Laboratoire de Mécanique des Solides (UMR CNRS 7649), Ecole Polytechnique  
F-91128 Palaiseau Cedex, France

bellis@lms.polytechnique.fr, bonnet@lms.polytechnique.fr

**Key Words:** *Inverse Scattering, Topological Sensitivity, Time Domain, Adjoint Field Method*

### **ABSTRACT**

This communication addresses the application of topological sensitivity to the numerical solution of cavity identification in elastic media. The topological sensitivity analysis arises in connection with the investigation of the asymptotic behaviour of the featured cost functional (here introduced as a means of formulating the inverse problem in terms of a minimisation) with respect to the creation of a cavity of infinitesimal radius and prescribed location in an otherwise cavity-free solid. Initially developed for the topological optimization of structures, this method provides a non-iterative computational tool for constructing a reliable void indicator function, as previously discussed in e.g. [1,2].

The cost functional used here is classically based on exploiting data about the boundary traces of the mechanical fields arising in wave-imaging processes. It quantifies the gap between quantities (e.g. displacements) based on a trial topology domain and on a reference domain. In practice, the reference quantities can be provided by experimental measurements or by numerical simulations. Such problems involve naturally integral formulations. The framework of the topological derivative, ie owing to the infinitesimal size of a cavity, of general functionals is presented in [3] in the linear elasticity case. More details can be found in [2] on the mathematical developments which leads to an analytical first order asymptotic expansion of cost functionals in a frequency domain. The results presented here use the derivation technics based on the use of an adjoint state. This method allows to deal with the topological gradient of general functionals with high simplicity and efficiency.

Our aim is to illustrate the efficiency of such non-iterative identification technique implemented in a conventional computational framework (here, the classical displacement-based finite element method together with a Newmark time-stepping algorithm). Results of numerical experiments will be presented for 2-D and 3-D time-domain elastodynamic cases, based on topological sensitivity formulas given in [1], in order to demonstrate the efficiency of the approach. Dynamical simulations will highlight the mechanisms underlying identification methods based on topological sensitivity. As well as other methods such as the linear sampling method [4] (not yet implemented for time-domain problems, to the best of our knowledge), such approach is demonstrated through numerical experiments to provide qualitatively good identification results while being computationally much more economical than ordinary, iterative, inversion procedures.

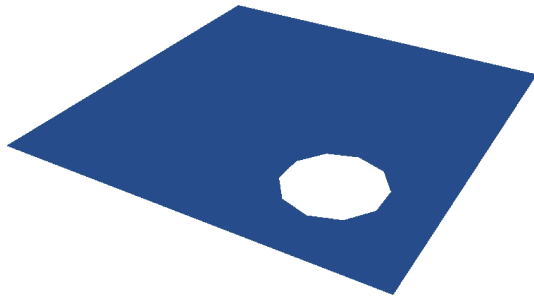


Figure 1: 2D elastic body with cavity

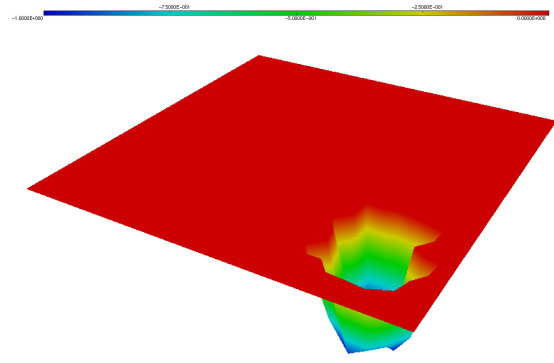


Figure 2: Thresholded topological sensitivity field (the most negative values indicating the locations where the nucleation of a small cavity induces the sharpest decrease of the featured cost function).

## REFERENCES

- [1] M. Bonnet. "Topological sensitivity for 3D elastodynamic and acoustic inverse scattering in the time domain". *Comput. Methods Appl. Mech. Engrg*, Vol. **195**, 5239-5254, 2006.
- [2] M. Bonnet and B. B. Guzina. "Sounding of finite solid bodies by way of topological derivative". *Int. J. Numer. Meth. Engng*, Vol. **61**, 2344-2373, 2004.
- [3] S. Garreau, P. Guillaume and M. Masmoudi. "The topological asymptotic for PDE systems : the elasticity case". *J. Control Optim.*, Vol. **39**, 1756-1778, 2001.
- [4] S. Nintcheu Fata and B. B. Guzina. "Elastic scatterer reconstruction via the adjoint sampling method". *J. Appl. Math.*, Vol. **67**, 1330-1352, 2007.