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Optimal shape design subject to variational inequalities

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ABSTRACT

In this talk, a shape optimization problem subject to an elliptic variational inequality is considered. A typical model problem is

$$\text{minimize } J(\Omega) = \int_\Omega (u(\Omega) - u_d)^2 dx \text{ over } \Omega \in \mathcal{O}_k := \{\Omega \subset D : \Omega \text{ of class } C^k\}$$

$$\text{subject to } |\Omega| \geq \epsilon > 0,$$

$$u(\Omega) = \operatorname{argmin}\left\{\frac{1}{2}\|\nabla u\|^2_{L^2(\Omega)} - (f,u)_{L^2(\Omega)} : u \in H^1_0(\Omega) \text{ with } u \leq \psi \text{ a.e. in } \Omega\right\},$$

where u_d is given data, D is the hold-all domain, f is a given volume force and ψ denotes a given obstacle with $\psi|_{\partial\Omega} > 0$. The first order optimality condition of the minimization problem in the constraint set is given by the variational inequality (VI)

$$\langle -\Delta u(\Omega) - f, u - u(\Omega)\rangle_{H^{-1}(\Omega),H^1_0(\Omega)} \geq 0 \quad \forall u \leq \psi, u \in H^1_0(\Omega). \tag{1}$$

Due to VI constraint the dependence of J on Ω is nonsmooth such that a *conical* Eulerian (semi)derivative has to be employed for studying shape sensitivity.

First we analyse the shape sensitivity of J. For this purpose we use a primal-dual version of (1), i.e.,

$$-\Delta u(\Omega) + \lambda(\Omega) = f, \quad u(\Omega) \leq \psi, \quad \lambda(\Omega) \geq 0, \quad \langle \lambda(\Omega), u(\Omega) - \psi\rangle_{H^{-1}(\Omega),H^1_0(\Omega)} = 0. \tag{2}$$

As a result we obtain a nonsmooth version of the usual shape gradient by considering special perturbation direction within a particular cone. In a next step we study topological sensitivity of J with respect to changing the topology of Ω by "drilling" a small ball-shaped hole in Ω. For this we rely on a smoothed version of (2). In fact, under additional regularity assumptions $\lambda(\Omega) \in L^2(\Omega)$. Then (2) is equivalent to

$$-\Delta u(\Omega) + \lambda(\Omega) = f, \quad \lambda(\Omega) = \max(0, \lambda(\Omega) + \sigma(u(\Omega) - \psi))$$

for arbitrarily fixed $\sigma > 0$. The smoothed version is obtained by employing a local C^1-smoothing of $\max$. Then we study the limit of the topological derivative as the regularization parameter tends to

zero. This step contains an asymptotic analysis of the solution of the VI with respect to the radius of the hole. We point out that particular analytical difficulties in the above considerations (especially for shape sensitivity) are connected to the so-called biactive set, i.e.,

$$\mathcal{B} = \{u(\Omega) = \psi \ \wedge \ \lambda(\Omega) = 0\}.$$

Finally, our analytical approach is turned into an algorithmic scheme for solving the model problem. The talk ends by a report on numerical results illustrating the efficiency of the topological sensitivity in this context. Further we highlight an application of our concepts to the electrochemical machining problem.