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Control-in-the-coefficient problems subject to variational inequalities

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ABSTRACT

In this talk, we study a class of optimal control problems subject to parabolic variational inequalities. This problem class is interesting for several reasons. (i) Problems of the aforementioned-type belong to the class of mathematical programs with complementarity constraints. These problems are challenging from the optimization-theoretic point of view as all classical constraint qualifications for the existence of (Lagrange) multipliers for characterizing stationarity are violated. As a result there is no longer a single stationarity concept. Rather one has to deal with various concepts such as weak, C-, B- or strong stationarity, for instance. (ii) The design of efficient numerical schemes is challenging due potential combinatorial aspects of the feasible set and the large scale of the discretized problems. (iii) Many important applications ranging from elastohydrodynamic lubrication to calibration problems mathematical finance are covered by this problem class.

In order to overcome the problem concerning constraint qualifications we first relax the feasible set. The resulting problem now satisfies classical constraint qualifications, but it turns out to be a state constrained optimal control problem. The latter problem class is challenging from the numerical point of view as the Lagrange multiplier associated with the state constraint typically is of low regularity. As a remedy we propose a Moreau-Yosida-based regularization scheme. The resulting PDE-constrained optimization problem can be treated by standard theory. For theoretical purposes we then study the limiting behavior as the above regularization parameters tend to zero. Mild assumptions on the regularization parameters yield so-called C-stationarity for the limiting problem. Under certain regularity requirements on the biactive set we further derive strong stationarity.

Following our proof concept in function space we design a numerical scheme for solving the discretized first order system. the Moreau-Yosida regularization provides a primal-dual path-following framework which was applied successfully earlier for solving obstacle-type problems and state constrained optimal control problems for PDEs. The talk ends by a report on numerical tests including parameter identification problems for American put-options.