

## **AUTOMATION OF THE PROCESS OF LIMIT ANALYSIS OF PRESSURE VESSELS WITH CAD/CAE TECHNOLOGIES - A new Shell Element to Include Bending and Membrane Effects**

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### **ABSTRACT**

This work shows the use of CAD/CAE technologies for the automation of the process of limit analysis of pressure vessels. The process was divided into the following phases: conception of the geometrical and discrete models referred to as **pre-processing, analysis and post-processing**. To perform the automation of the process the phases were implemented in two integrated modules of a computational platform developed under the paradigm of object oriented programming (OOP), which implies in modularization and easy extensibility of the code. The geometrical modeling, the finite element discretization and the graphical representations of the solutions, including the collapse mechanism, are executed by a CAD system developed for this purpose. The analysis is performed by a CAE system developed using C++, where classes and relationships were constructed under OOP to implement the new finite element formulation. Automation of the process is achieved by integrating the CAD and CAE systems. The CAD system was constructed as a plug-in of the software AutoCAD using the Application Program Interface-API called ObjectARX (AutoCAD Runtime eXtension). Such an API allows the customization of the graphical environment and more important permits the creation of new entities, which can inherit all the functionality of AutoCAD native classes and yet have their own attributes. Those classes can store the necessary data for the computational modeling of structural elements, which can be abstracted for a perfect 3D representation and for different type of structural analysis. The finite element formulation of limit analysis problems of symmetrically loaded pressure vessels is computed by an improved adaptive technique implemented in the CAE system. A new finite thin shell element has been proposed, which can model bending and changes of curvature along the element as well as the hinge behavior at the nodes. The element is also capable of simulating membrane behavior. The element was implemented into the finite element technique proposed by Franco and Ponter [1,2] to produce upper bound estimates on limit loads of pressure vessels. The kinematic technique is reviewed and generalized to incorporate the new element displacement. The improved formulation is associated to a new piecewise linear (PWL) yield surface which, constitutive laws capable of simulating the variation of curvature within the element during collapse can be extract from. The PWL constitutive laws are extracted from the ten planes yield surface in Fig. 1(c) generated by linearizing the exact yield

surface, Fig. 1(a), following the procedures in Drucker [3]. These yield conditions are much closer representation of the exact yield surface (see Onat [4]) when compared to the hexagonal prism, Fig. 1(b), used in Franco and Ponter [1,2]. The new element and yield surface allowed the computation of the energy dissipation rate due to the change in the curvature within the element during collapse. Although the plastic hinge formation is still restricted to the nodes volumes, membrane and bending behaviors are captured simultaneously within the element volume by the new yield surface. The rate of dissipation of energy during collapse is computed by considering the variation of curvature  $\kappa_\phi$  along the elements volumes and the rates of membrane strain  $\varepsilon_\phi$ ,  $\varepsilon_\theta$  within the elements need also to be considered and are assumed to be plastic only. A systematic procedure to find a consistent relationship between nodal displacements at collapse and the constitutive nodal plastic multipliers of the PWL yield conditions allowed the reduction of the plastic analysis to a Linear Programming (LP) problem. The collapse mechanisms show the effect of bending inside the element, which indicates more realistic solutions. The formulation was applied to calculate upper bounds estimates at collapse for internal pressure and ring loads applied to cylindrical vessels. The results obtained show better, ie. smaller upper bounds, when compared to other numerical and analytical solutions. Finally, an *a posteriori* error estimator had to be re-written to include the new displacement rate and PWL strain rate fields based upon which an adaptive scheme was used to refine the finite element mesh.

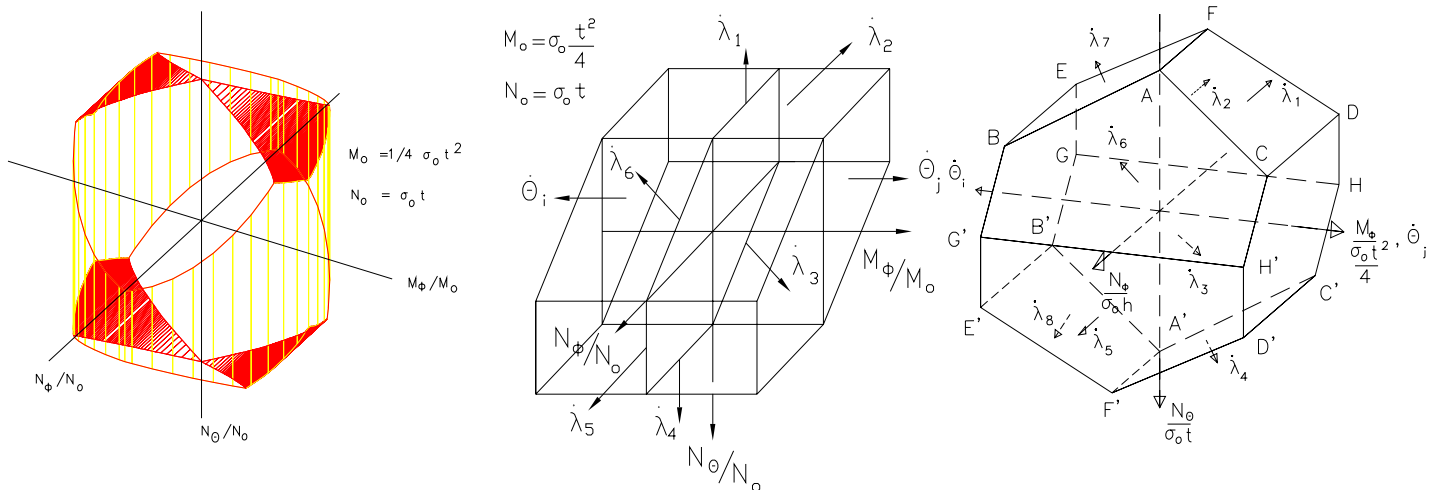


Fig. 1 (a) Exact yield surface (b) Circumscribed hexagonal prism yield surface. (c) Ten planes yield surface.

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