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A NUMERICAL ADAPTIVE ALGORITHM COMBINING DOMAIN DECOMPOSITION

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ABSTRACT

This paper is devoted to an adaptive finite element method for elliptic problems using domain decomposition techniques. Adaptive mesh refinement based on a posteriori error estimates is an essential instrument for efficient numerical solving of PDES. Domain decomposition methods are very useful when the original problem has different features on distinct regions of the initial domain, such as the nonlinearity. A lot of work has been made (see,e.g, [1]) in this area and there are many applications in diverse fields (see,e.g, [2]). We construct an adaptive finite element method for elliptic problems (adaptive modified Uzawa method) using domain decomposition. Therefore, we can consider different initial triangulation and an independent adaptive mesh refinement in each subdomain. We are going to consider a linear stationary problem defined in a Ω domain, decompose the domain into two subdomains Ω_1 y Ω_2, Γ_{12} is the interface, and we apply on each subdomain an adaptive element finite method (A.F.E.M.) using mesh refinement based on a posteriori error estimative. Under these assumptions it is not difficult to generalize the algorithm to the nonlinear case. The convergence is proved with respect to a discrete solution in an space corresponding to a sufficiently refined mesh. We get the following convergence result: Let *Sea* (U_j, P_j) *be the sequence of solutions obtained by the adaptive modified Uzawa algorithm. There exist positive constants* C *and* $\delta < 1$ *such that*

$$\|u - U_j\|_{\mathbb{V}} + \|p - P_j\|_{\mathbb{M}} \leq C\delta^j$$

where $\mathbb{V}$ and $\mathbb{M}$ are proper functional spaces. u and p are the discrete solutions on a sufficiently refined mesh.

The starting point is the **Hybrid Primal formulation** of an elliptic problem : Find $(u,p) \in \mathbb{V} \times \Lambda$ such us

$$\sum_i \int_{\Omega_i} \mathbf{a}\nabla u \nabla v + \sum_i \int_{\partial\Omega_i} pv = \sum_i \int_{\Omega_i} fv \quad \forall v \in \mathbb{V} \tag{1}$$

$$\int_{\Gamma_{12}} [u]\mu = 0 \qquad \forall \mu \in \mathbb{M} \tag{2}$$

with $i = 1, 2$ and $[u] = u_1 - u_2$. It is weel known that the rate convergence of the Uzawa algorithm is very low to solve this kind of problems. In this work we modify the Uzawa algorithm in two ways: First we will use different auxiliary operators to solve the equation 2 in order to accelerate convergence. Second, we introduce mesh adaptivity (Adaptive Modified Uzawa algorithm) The algorithm is described below:

Choose parameters $\rho > 0$ such us $\sigma := \|I - \rho S\|_{\mathcal{L}(\Lambda,\Lambda)} < 1$, $0 < \gamma < 1$ and $\varepsilon_0 > 0$; set j=1.

1. Given a finite space $\mathbb{V}_0$ and an initial approximation $P_0 \in \mathbb{M}_0$.
2. Compute U_j on $\mathbb{V}_j$.
3. Update P_j on $\mathbb{M}_j$ using P_{j-1} and U_j.
4. $\varepsilon_j \leftarrow \gamma\varepsilon_{j-1}$.
5. Compute T_j by adapting the mesh T_{j-1}, such that $|U_j - u| < \varepsilon_j|$
6. $j \leftarrow j + 1$.
7. Go to step 2.

Here S is the Schur complement associated to the Uzawa algorith. The pair (U_j, P_j) is the discrete solution for an approximated problem. We will show some numerical results obtained using the AMUADD algorithm, namely problems with some singularity in a well defined region of the original domain. Hence we could do a decomposition domain isolating the singularity on a given subdomain. Numerical tests show that the method could be adapted to nonlinear problems. In booths cases the computation time and the computational resources are less than without domain decomposition. To conclude we may say that AMUADD is suitable to use on parallel machines.

REFERENCES

[4] J.L. Lions and O. Pironneau. “Non-overlapping domain decomposition for evolution operators”. *C. R. Acad. Sci. Paris.*, t. **330**, Sér. **I**, 943–950, 2000.

[5] R. Glowinski. *Numerical methods for fluids*, 1th. Edition, Vol. **IX**, P.G.Ciarlet and J. L. Lions Editors, 2003.