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AN OPTIMIZED EXTRAPOLATION SOLUTION FRAMEWORK FOR PARABOLIC PDEs

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ABSTRACT

The simplest method used to improve the accuracy of a numerical solution with a limiting computational cost is called Richardson extrapolation (RE). It applies to many different discretization frameworks and is fairly easy to implement. Its use in Computational Fluid Dynamics (CFD) [1, 2, 3] is rather popular and has brought a lot of attention. Among the variations of the RE method that have been studied are

- approximate a fine grid solution instead of an asymptotic limit,
- works with non embedded grids [3],
- retrieve the convergence order of the method if it is an unknown [4].

All these extensions rely on the a priori existence of an asymptotic expansion of the error such as a Taylor formula, and make no direct use of the PDE formulation. As a consequence RE methods are extremely simple to implement. But in practice, meshes might not be fine enough to satisfy accurately the a priori convergence estimates that are only asymptotic in nature. RE is then unreliable. From the numerical analysis of RE extrapolation formula, one can see that these technique are fairly unstable, and sensitive to noisy data [5]. To cope with these limitations of RE, Garbey and Shyy have introduced recently [5, 6] the so-called Least Square Extrapolation method (LSE) that is based on the idea of finding automatically the convergence order of a method as the solution of a least square minimization problem on the residual. The LSE method is based on the post-processing of data produced by existing PDE codes. The method has been described in detailed in [5]. From a practical point of view, the authors have used a two dimensional turning point problem exhibiting a sharp transition layer as well as a finite difference approximation of the cavity flow problem to show that the LSE method is more reliable than RE while the implementation is still fairly easy and the numerical procedure inexpensive. Garbey and Shyy have recently extended the framework to a general optimized extrapolation solution (OES) method that provides error estimates for arbitrary norm [6].

In this paper, we focus on improving the accuracy for solutions of *parabolic* problems using OES. The difficulty is to deal with space and time together in the extrapolation formulation.

There are a number of interesting papers on *a posteriori* estimate for unsteady problems [7, 8, 9]. However, most often unsteady problems

$$\frac{\partial u}{\partial t} = N[u], \tag{1}$$

are analyzed in their semi-discretized form

$$-dtN[u] + u = F, \tag{2}$$

where dt is the time step, to reuse the same *a posteriori* framework than for the steady problem.

We propose to use our optimized extrapolated solution method to produce error estimate using grid solutions that can be produced by various discretization methods. This approach might be combined to existing *a posteriori* estimate when they are available, but is still applicable as a better alternative to straightforward RE when such stability estimate are unavailable.

In this paper, we pursue the research initiated in [5] to generalize the framework to parabolic problems. We use then OES with coarse grid solutions that have different meshes in space *and* time. This is therefore more than a simple extension of the previous OES method to the semi-discretized problem (2). We will investigate the power and limit of this new OES with a broad variety of non-linear parabolic problems.

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