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Simple A Posteriori Error Estimators for the h-Version of the Boundary Element Method

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ABSTRACT

The h-$h/2$-strategy is one very basic and well-known technique for the a posteriori error estimation for Galerkin discretizations of energy minimization problems. Let ϕ denote the exact solution. One then considers

$$\eta_H := \|\phi_h - \phi_{h/2}\|$$

to estimate the error $\|\phi - \phi_h\|$, where ϕ_h is a Galerkin solution with respect to a mesh $\mathcal{T}_h$ and $\phi_{h/2}$ is a Galerkin solution for a mesh $\mathcal{T}_{h/2}$ obtained from uniform refinement of $\mathcal{T}_h$. We stress that η_H is always efficient – even with known efficiency constant $C_{\text{eff}} = 1$, i.e.

$$\eta_H \le \|\phi - \phi_h\|.$$

Reliability of η_H follows immediately from the assumption $\|\phi - \phi_{h/2}\| \le \sigma\,\|\phi - \phi_h\|$ with some saturation constant $\sigma \in (0,1)$. Under this assumption, there holds

$$\|\phi - \phi_h\| \le \frac{1}{\sqrt{1-\sigma^2}}\,\eta_H.$$

However, for boundary element methods, the energy norm $\|\cdot\|$ is non-local and thus the error estimator η_H does not provide information for a local mesh-refinement. Recent localization techniques from [1] for $\widetilde{H}^{-\alpha}$-norms and [3] for $\widetilde{H}^{\alpha}$-norms allow to replace the energy norm in this case by h-weighted L^2-norms resp. H^1-norms, where h denotes the local mesh-size. In particular, this very basic error estimation strategy is also applicable to steer an h-adaptive mesh-refinement. For instance, for Symm's integral equation, the L^2-norm based estimator

$$\mu_H := \|h^{1/2}(\phi_h - \phi_{h/2})\|_{L^2(\Gamma)}$$

is equivalent to η_H. We thus may use μ_H to steer the mesh and η_H to estimate the error.

Further simplifications of the proposed error estimators η_H and μ_H consist of replacing ϕ_h by some appropriate projection $\Pi_h \phi_{h/2}$, for instance, by use of the L^2-projection onto the discrete space corresponding to $\mathcal{T}_h$.

Moreover, the error estimator η_H is proven to be equivalent to the averaging estimator in [4] and the two level estimator from [5].

Numerical experiments in 2D and 3D for first-kind integral equations with weakly-singular integral operator conclude the talk.

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