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A NUMERICAL COMPARISON OF TWO THERMOVISCOELASTIC CONSTITUTIVE EQUATIONS BASED ON THEIR TEMPERABILITY CAPTURING CAPABILITY

***N. Troyani[1], Y. Ulacio[1], P. Baíz[2] and C. Gomes[1]**

[1]Centro de Métodos Numéricos en Ingeniería, Universidad de Oriente, Puerto La Cruz, Venezuela
ntroyani@cantv.net

[2]Department of Aeronautics, Imperial College, London, UK,
p.m.baiz@imperial.ac.uk

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ABSTRACT

It is a well known fact (confirmed experimentally) that during sudden cooling (tempering) of a thermoviscoelastic medium, some residual stresses are set in the material once a total cool down process is completed. These residual stresses can be beneficial in some applications [1] by increasing fracture strength, and detrimental in others [2] where signal transmission may be impaired. Typically, during the stated cooling from a sufficiently high temperature (at which the whole member exhibits a predominantly viscous behaviour) the surface material and outer regions material of a given member will reach a predominantly elastic behaviour sooner than the core material of the member, wherein predominantly viscous behaviour will last longer due to the stated lagged cooling [3]. That is, the transient temperature fields will impose different material behaviour in different parts of the member depending on the specific temperature evolution in each part resulting in both different relaxation rates (viscous effects), and setting of the stated residual stresses once the temperature field becomes uniform in the long term. As described in [3] the magnitude of the stated residual stresses represent a measure of the temperability of thermoviscoelastic materials and thus provide an appealing, physically induced, way to test constitutive equations for the mechanical modelling of thermoviscoelastic materials. Further, it was stated therein "that such constitutive models must be able to correctly capture this phenomenon in the sense that "computed residual stresses must reasonably approximate experimentally determined results". One distinguishing feature of viscoelastic materials is the fact that their present behaviour depends, not only, on current stresses (strain), depending on the type of utilised formulation, it depends also on past stress (strain) history. Having proposed the concept that capturing temperability constitutes a valid test for thermoviscoelastic constitutive hypotheses [3], in this work we compare two published

thermoviscoelastic constitutive equations (that comply with the requirements of the class of thermorheologically simple materials, TRS) in terms of the residual stress magnitude that results from rapid cooling of an infinite cylinder of Float Glass. Specifically, we propose a comparison of Moreland and Lee constitutive hypothesis [4], and that of Stouffer and Wineman constitutive hypothesis [5], in terms of their prediction of the stated residual stresses, by assuming certain values of the scaling parameters incorporated in the later theory. The calculations are performed using two identical FORTRAN finite element based codes that differ only on the utilised constitutive equations. Both codes are framed on a principle of virtual work scheme and as a result equilibrium is satisfied at each computational step. The geometry used for the test is that of an infinite cylinder of float glass (plane strain condition in the context of a small deformation theory), a material known for its sensitivity to develop residual stresses during tempering. Due to symmetry, only a small wedge of the cylinder is used for the finite element model as indicated below in Figure 1.

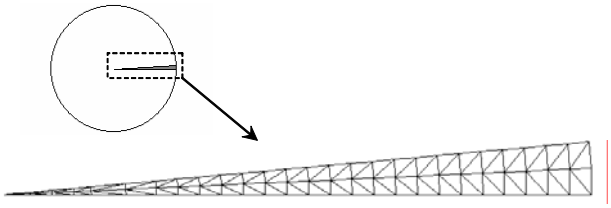

Figure 1. Transversal cross section of the infinite cylinder of float glass showing thin wedge sub domain and finite element computational domain.

The thermoviscoelastic material characterization was done through an experimentally determined relaxation function and an experimentally determined time-temperature shift function. The results indicate that on one hand, both theories predict reasonably well the setting of residual stresses in the glass cylinder, and at the same time Stouffer and Wineman's hypothesis offers the possibility of capturing additional residual stresses effects.

References

[1] P. Baiz and N. L. Troyani, Size dependency of the temperability of thermoviscoelastic materials, *Technical Bulletin IMME*, Vol. **40** (2), pp. 14-20, 2002.

[2] A. D. Yablon, Optical and mechanical effects of frozen-in stresses and strains in optical fibers, *IEEE Journal on Selected Topics in Quantum Electronics*, Vol. 10 (2), pp. 300-311, 2004.

[3] N. L. Troyani, C. Gomes, P. Báiz, Capability for capturing temperability as a criterion for the validity of thermoviscoelastic constitutive equations. *CMM-2007, Computer Methods in Mechanics*, Lodz-Spala, Poland, 2007.

[4] L. W. Moreland and E. H. Lee, Stress analysis for linear viscoelastic materials with temperature variation, *Transactions of the Society of Rheology*, V. IV, pp. 233-263, 1960.

[5] D. C. Stouffer and A. S. Wineman, Linear viscoelastic materials with environmental dependent properties, *Intl. J. of Engineering Science*, Vol. 9, pp 193-212, 1971.