DETECTION OF OBSTACLES WITH A NEW PIEZOELECTRIC PATE MODEL

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ABSTRACT

We present a new frictional contact piezoelectric plate model and show how it can be used for the detection and study of obstacles.

The derivation of the model (cf. [1]) is done by applying an asymptotic method to the three-dimensional equations describing a thin anisotropic plate in contact with a rigid obstacle. The contact is unilateral, i.e., the contact region is not known in advance. It is modelled by the classical Signorini contact conditions. For the frictional behavior of the plate, the Tresca friction law is used.

In the asymptotic process, the thickness of the piezoelectric plate is driven to zero and the convergence of the unknowns (the mechanical displacement and electric potential of the plate) is studied. This convergence process leads to two-dimensional modified Kirchhoff-Love plate equations, in which mechanical displacement and electric potential get partly decoupled. To be precise, the electric potential can be derived as an explicit function of the mechanical displacement.

Based on the asymptotic model we show numerical simulations that illustrate how the direct piezoelectric effect of the plate can be used to detect obstacles. The two-dimensional asymptotic model, which is defined in the middle plane of the plate is discretized using finite elements; The numerical treatment of the contact conditions for the mechanical displacements follows [2].

We use a plate with a square middle plane which is clamped along its left lateral side. It is made of two different layers of PZT piezoelectric ceramic materials (the material parameters are taken from the tables VIII and XI in [3]). The plate is subject to gravity force and there are no applied electric charges. We suppose that a zero electric potential is imposed on the lower surface and on a part of the lateral boundary of the plate. On the upper surface of the plate, we assume homogeneous electric Neumann boundary conditions. The plate may be in contact with a rigid obstacle on its lower surface. Due to the gravity force, the plate deforms from its original state, possibly getting in contact with the obstacle. Figure 1 shows various obstacles, the corresponding deformed plates and contour plots of the electric potentials measured on the upper surface of the plate. In these examples we neglect the frictional forces and only assume Signorini contact conditions. Note that the electric potential on the upper surface



Figure 1: Different obstacles (left), the corresponding deformed plates (middle) and contour plots of the electric potentials on the upper surface of the plate (right).

allows to guess the shape of the obstacle. Thus, these tests demonstrate the appropriateness of the new asymptotic model and a useful application for thin anisotropic piezoelectric plates: they can be used as sensor devices for scanning and recognizing unknown surfaces.

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