8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

PARAMETERS IDENTIFICATION UNDER UNCERTAINTIES OF INTERVAL TYPE

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Key Words: *Parameters identification, Inverse Problem, Uncertainties, Applications.*

ABSTRACT

In this work the problem of construction of a mathematical model of a stationary process under uncertainties of interval type is examined [1,2]. It is supposed that the amount of measurements for each variable is minimal.

For simplicity of argument, let us consider only the problem of construction of a linear model (linear regression)

$$q_1 = z_1 q_2 + z_2 q_3 + \ldots + z_{n-1} q_n + z_n, \qquad (1)$$

where $z_1, z_2, \ldots, z_n$ are unknown coefficients of the stationary mathematical model, the chosen characteristics of process $q_1, q_2, q_3 \ldots, q_n$.

It is supposed that for every variable q_i ($i = 1,.., n$) we have n measurements q_{ik} ($k = 1, 2, \ldots, n$). This problem in classical formulation can be reduced to a linear non-uniform system of algebraic equations which is most commonly solved by the method of least squares [3,4].

In the present work the problem of synthesis of parameters of multivariate regression is considered in which the information about statistical properties of measurements are not used.

As the measurements of variables are received experimentally it is assumed that each measurement q_{ij}, $1 \le i, j \le n$ has some error the maximal size of which is known (interval type of uncertainties):

$$| q_{ij} - q^{ex}_{ij} | \le \delta_i , \ 1 \le j \le n, \ i = 1,2, \ldots, n. \qquad (2)$$

where q^{ex}_{ij} is exact measurements of variable q_i.

The statistical characteristics of errors of measurements are unknown.

The problem of construction of mathematical model of stationary process under uncertainties of interval type is reduced to the solution of linear algebraic system with inexact matrix of system and inexact right part:

$$A_p z = u_{\delta_1}, \qquad (3)$$

where $u_{\delta_1} = q_1$; $u_{\delta_1} \in U = R^n$; $z \in Z = R^n$; $\| A^{ex} - A_p \|_{Z \to U} \le h$, A^{ex} – exact matrix of (3); $\| u_{\delta_1} - u_1^{ex} \| \le \delta_1$, u_1^{ex} – exact right part of (3); $u_1^{ex} = q^{ex}_1$, $u_1^{ex} \in U = R^n$; $\| . \|$ is the norm of a vector in Euclidean space R^n; $A^{ex} z = z_1 q_2^{ex} + z_2 q_3^{ex} + \ldots + z_{n-1} q_n^{ex} + z_n e$, q_i^{ex} – exact measurements of vector q_i, $i = 1,2,\ldots n$.

Let us denoted vector p as vector from space $R^n \oplus R^n \oplus R^n \oplus \ldots \oplus R^n = R^{(n-1)n}$:

$p^T=(q_{21}, q_{22},...,q_{2n}, q_{31},q_{32},...,q_{3n} ,..., q_{n1},q_{n2},...,q_{nn})$, $(.)^T$ is the sign of transposition. Each vector q_i can accept meanings in some closed area $D_i \subset R^n$ by virtue of inexactness of q_{ij}. Vectors p can accept meanings in some closed area $D=D_1 \oplus D_2 \oplus D_3 \oplus ... \oplus D_n \subset \subset R^{(n-1)n}$. The certain operator A_p associates with each vector p from area D. The class of operators $\{A_p\}=K_A$ will correspond to the set $D \subset R^{n(n-1)}$.

It is supposed that among matrixes $\{A_p\}$ exist the matrix A_p^0 with det $A_p^0 = 0$ with any value of h. It is easy to show that the problem (3) is incorrect problem under these conditions.

Some variants of problem formulation of parameters identification are considered: the estimation from below of possible solutions, solution that has the greatest norm amongst of solutions with minimal norm, the more plausible solution of problem and so on [2,5].

The modification of Tikhonov's regularization method was used for solution incorrect problem (3) [6].

For example, the more plausible solution $z_{\delta_1}^{pl}$ of problem (3) is the solution of the following extreme problem

$$\left\| A_{p^{opt}} z_{\delta_1}^{pl} - u_{\delta_1} \right\|^2 = \inf_{z_a} \sup_{A_p \in K_A} \left\| A_p z_a - u_{\delta_1} \right\|^2, \qquad (4)$$

where z_a is the solution of extreme problem with different vectors a from D.

$$\| z_a \|^2 = \inf_{z \in Q_{\delta_1, a}} \| z \|^2. \qquad (5)$$

The set $Q_{\delta_1,p}$ is defined as $Q_{\delta_1,p} = \{ z : \| A_p z - u_{\delta_1} \| \leq \delta_1$.

The calculations on real measurements were executed for comparison with known methods.

The offered approach to a problem of identification of parameters of static linear mathematical model (multivariate regression) allows expanding of class of the possible solutions up to maximum possible.

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