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Adaptive wavelet method for solving stochastic convection-diffusion equation

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ABSTRACT

The transport and diffusion of ingredient in a flow field is a basic process in modeling air/water pollution propagation. This process is governed by the convection-diffusion equation and subjected to random fluctuations[1]. Here we consider such stochastic convection-diffusion equation in the concentration field under uncertain inputs, i.e. random flow (transport) velocity or/and source (forcing) term. In dealing with the uncertainty involving in the partial differential equation, we first expand the random functions in terms of the truncated polynomial chaos, then perform Galerkin projection of the equations on the polynomial basis[see e.g. [2-5]. This procedure leads to a coupled deterministic equation system for the coefficients of the expansion. Once the coefficients of the expansion are obtained, the spatio-temporal variations of the concentrations have been specified in terms of the polynomial chaos expansion, from which statistical moments, i.e. the expectation (mean value), variance and higher moments, can be readily computed.

In [5], we developed a finite difference V-cycle multigrid solver to iteratively solve the system on different levels of mesh. Our numerical study shows that simulations based on the probabilistic modeling provide valuable information (e.g. safety factors) for decision making in the engineering design. The cost paid here is the increased size of the equation system that needs to be solved. This is caused by the spectral representation, which gives each computational grid point $P + 1$ unknowns: C_k $(k = 0, \cdots, P)$, where $P + 1 = (n + p)!/(n!p!)$, and n is the random variable dimension, p is the order of the polynomial chaos expansion. Thus, the size of the system is much larger than it in the deterministic case, and increases rapidly with n and p, that poses a serious computational challenge in its practical engineering applications. The efficiency of the standard multigrid solver we used needs to be improved based on exploiting the solution structure of the stochastic equation system.

Here, we observe that the unknowns C_k $(k = 0, \cdots, P)$ on each computational grid point represent fluctuations of different scales. Thus solution of the equation system is "*grid-wise*" multiscale in nature

and traditional *mesh-refinement* is not applicable. In this work, we utilize the excellent properties of the wavelet in non-linear approximation theory[6] to design and construct an adaptive wavelet collocation solver[7,8]. The refinement for the adaptation is driven by selection of the basis functions. This leads to a "*space-refinement*" procedure, that enables the $P + 1$ unknowns at each grid point to be naturally adapted independently of the rest, i.e. every unknown has its own active grid. In the time-dependent evolutionary case, the adaptive process is *dynamical* in the sense that the wavelet decomposition of the current available solution suggests a sparse, non-uniform active grid for the wavelet expansion of the solution at the next step. Moreover, the adaptation requires no projection in the data transfer between old and new meshes. The intermediate numerical results can be updated in a straightforward manner. Therefor it enjoys great advantages over the traditional re-meshing procedures. Further, due to the hierarchical structure and compact support feature of the wavelet bases, less computations involved in the reassembling of matrices at each refinement – rendering a more efficient solver.

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