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MOTION OF A CAPSULE IN A SIMPLE SHEAR FLOW: EFFECT OF MEMBRANE DESCRIPTION AND BENDING STIFFNESS

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ABSTRACT

Bioartificial capsules consisting of an internal liquid protected by a thin membrane have potential therapeutic uses, such as drug delivery in the microcirculation and artificial blood design. Numerical models are necessary to predict their behaviour in microflow conditions encountered during fabrication and transport. This problem involves strong fluid-structure coupling where the membrane is undergoing large deformations due to the viscous flow of the internal and suspending liquids.

Numerical models have already been developed, describing the capsule wall as a thin hyperelastic membrane with stresses constant through the thickness. However, in certain flow conditions, such as simple shear flows at low shear rates and parabolic flows, compressive tensions appear in the membrane of the capsule, leading to wrinkling instability [1]. A more realistic model involving stress variations through the thickness of the capsule wall is needed to take the bending stiffness into account and to determine the membrane post-buckling behaviour.

The objective of this study is to couple a finite element model to describe the capsule wall in large deformation with a boundary-integral formulation of the Stokes equations that describe the flow. In this model, the equilibrium of the capsule is expressed in weak form using the virtual work principle. This approach thus differs from the methods commonly used so far in capsule studies, which rely on the local equilibrium equations. We first implement the method for a hyperelastic membrane.

The membrane is meshed using constant strain triangle elements (CST), with three degrees of freedom (the components of the displacement vector) at the three nodes. At a given time step, the elementary internal force vector is derived from the known displacement vector, using the Neo-Hookean law. The global internal force vector obtained by assemblage of all internal force vectors. The equilibrium of the membrane, expressed in its weak form, gives the external load at the nodes.

The Stokes equations are then solved on the same mesh using a boundary integral method, which computes the velocity at the nodes from the distributed external forces on the membrane. At the following

time step, the position of the nodes is updated by means of Lagrangian tracking through second-order Runge-Kutta integration.

As a benchmark study, we focus on the case of an initially-spherical capsule (radius a) in a simple shear flow (shear rate $\dot{\gamma}$). The internal and external fluids are identical Newtonian liquids (viscosity μ). The Reynolds number is assumed to be zero and the flow is therefore governed by the Stokes equations. The strength of the flow is given by the capillary number $Ca = \mu\dot{\gamma}a/G_s$, where G_s is the surface shear modulus of the membrane. This configuration has been extensively studied [1,2].

Using this method, we find results consistent with the previous studies. In particular, we find three different types of behaviours depending on the value of the capillary number. For moderate values of Ca, a stationary shape can be attained. For larger values, two tips appear on the membrane along the direction of elongation and the computation does not reach a stable shape. Finally, for small values of Ca, compressive tensions appear and lead to numerical wrinkles, with a wavelength depending on the size of the mesh. These wrinkles render the computation unstable.

In conclusion this numerical method solves an explicit fluid-structure interaction problem, which couples a finite element model with a boundary integral method to solve the Stokes equations. The advantage is that we need only to mesh the capsule wall. These preliminary results establish the feasibility of this coupling procedure. The next step is to take into account the bending stiffness. An approach is to discretise the capsule wall using the DKT12 discrete Kirchhoff triangular shell element [3], which results from the superposition of the CST membrane element used above and a triangular plate element (DKT6).

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