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Stress-constrained topology optimization by truly-mixed finite elements

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ABSTRACT

Minimum compliance topology optimization produces final designs that exhibit optimal stiffness performances without taking care of the stress layout in the material. Even if this procedure generally achieves topological shapes that are mainly made of bars or arch-fashioned members governed by regular stress patterns, final designs may be affected by the arising of stress concentrations in certain parts of the optimal domain. Classical examples refer to cornered regions, as in the well-known benchmark of the L-shaped lamina, or to the ground constraints zones in the domains to be optimized. In both the cases stress singularities generate several difficulties from the point of view of engineering and manufacturing of the pure minimum compliance optimal solutions.

To overcome these problems a set of local stress constraints may be introduced in the optimization problem in order to control the concentrations and to produce fully exploitable designs. A classical approach is the stress-constrained minimum weight problem tackled in [5], where the problem to minimize the weight of a bidimensional structure under an assigned material yielding limit is solved relying on traditional finite element schemes, as displacement-based discretizations. Moreover, within the imposition of local stress requirements, one has to take into account the so-called "singularity problem" and classically resort to ε-relaxation approaches [4] to reliably implement element-wise constraints.

The approach presented in this contribution is conversely based on the implementation of a set of local stress-constraints within a bidimensional minimum compliance setting, having the main aim of controlling the stress singularities above described that arise in a pure minimum compliance framework. Instead of the classical ε-relaxation, an alternative q-p approach is used to overcome the numerical difficulties related to the local imposition of stress limits. This procedure has been tested in [2], where it is used in conjunction with the computationally cheapest mixed fem scheme that descends from the variational principle of Hellinger-Reissner [3], exploiting a separate discretization of stresses and displacements to improve the accuracy of stress constraints requirements.

Two discretizations of the H-R variational principle may in fact be derived. The first one, used in [2], has continuous displacements as main variables and discontinuous

stresses as Lagrange multipliers. With respect to a classical displacement-based framework it has the main benefit of providing an independent interpolation of the stress field with the minimum increase in terms of computational burden. Alternatively, as presented in this contribution, one may rely on the second discretization of the H-R variational principle, often referred to as truly-mixed, that has regular stresses as main variables and discontinuous displacements as secondary ones. Among the few elements able to pass the "inf-sup condition" [3] under these conditions, the Johnson-Mercier element [6] is herein implemented. The JM triangle has fifteen degrees of freedom for stresses and has the peculiarity of allowing a remarkable improvement in the description of the stress field and therefore in the control of stress concentrations in optimal designs by means of the imposition of local stress constraints.

The contribution firstly presents the stress-constrained minimum compliance setting within a truly-mixed framework, with peculiar attention to the numerical schemes adopted to reduce the computational burden related to the adoption of a JM-based discretization. Afterwards classical and alternative numerical examples are introduced to test the capabilities of the implemented method and to compare its performances with respect to optimization schemes based on classical stress-constrained displacement-based schemes. Peculiar attention is moreover paid to the different mechanical behaviour peculiar to the optimal topology layouts achieved within different topological framework based on mixed or displacement-based discretizations, with or without stress-constraints.

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