8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Wave Splitting as a Tool for Inversion Algorithms and Analysis

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Key Words: *Inverse Problems, Object Identification, Wave Splitting, Uniqueness Techniques.*

ABSTRACT

We present recent results on wave splitting procedures for acoustic or electromagnetic scattering problems, compare [1] and [8]. The idea of these procedures is to split some scattered field u^s into a sum

$$u^s = u_1^s + u_2^s \tag{1}$$

of fields coming from different spatial regions such that this information can be used either for inversion algorithms or for active noise control.

Splitting algorithms can be based on general boundary layer potential representation or Green's representation formula. We will prove the *uniqueness* of the decomposition *of the scattered wave* outside the specified reference domain $G = G_1 \cup G_2$ and the uniqueness of the decomposition *of the far-field pattern*

$$u^\infty = u_1^\infty + u_2^\infty \tag{2}$$

with respect to different reference domains G, compare the Figure below. The techniques employed here are strongly related to the range-test as presented in [5] and the Kirsch-Kress method [3]. The splitting procedure is *not* an approximate method, but it describes a full decomposition of the field into two parts which are analytic outside of some subdomain G_1 or G_2 respectively, and satisfy the radiation condition.

In part two, we employ the splitting technique for field reconstruction for a scatterer with two or more separate components, by combining it with methods for wave recovery or other reconstruction schemes as presented in [2], [3], [4], [6] and [7]. Using the decomposition of a scattered wave as well as its far-field pattern, the wave splitting procedure gives an efficient way to reconstruct scattered waves near obstacles, from which the multiple obstacles which cause the far-field pattern can be reconstructed separately. This considerably extends the range of the decomposition methods in the area of inverse scattering. In particular, we will show recent results which could be achieved for the point source method [7]. We provide numerical examples to prove the feasibility of the splitting method.

Finally, for further applications of the splitting technique we also refer to recent work on magnetic tomography [8].

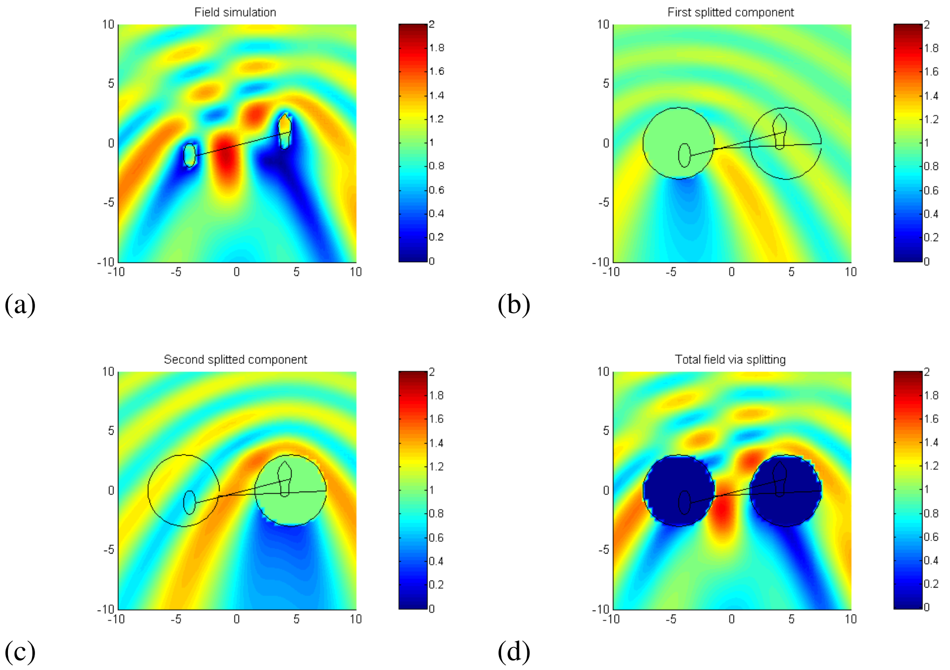

Figure 1: We show (a) the simulated total field for scattering by two scatterers. The figures (b) and (c) show the fields $|u_1^s + u^i|$ and $|u_2^s + u^i|$ and in (d) we visualize the modulus of $u_1^s + u_2^s + u^i$, i.e. the sum of the decomposed fields as test.

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