8th World Congress oh Computational Mechanics (WCCM8)
5th European Congress on Computational Mechanics and in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July , 2008
Venice, Italy

STABILIZED P1P0 FINITE ELEMENT APPROXIMATIONS OF COUPLED FREE SURFACE TO POROUS MEDIA SUBSURFACE FLOW PROBLEMS

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Key Words: *Coupled Flow, Surface and Subsurface Flows, Finite Element, Stabilization Method*

ABSTRACT

In many industrial filtration processes, viscous fluid is forced to flow into a porous media, with a free surface fluid flow as a control aspect. Industrial substances, in filtration, or pollution and contaminant, in environmental, could effects fresh water reserves [1]. Those effects are conditioned by free to porous media coupled flow. A better comprehension of such coupled flows is needed [2] for precisely quantify the industrial filtration efficiency and also to evaluate and manage the pollution or contaminant environmental damage impacts from human discharges in fresh water disposability. To adequately deals with the season behavior of environmental flows, and/or the invertible changing flows in industrial processes, a full transient formulation for such coupled free to porous media flows must be employed. Nevertheless, the interactions between free surface and porous media flow are not yet well understood. These aspects were only partially considered and not in full account by actual models to this time [3]. An efficient way to treat these problems relies on a computational modeling approach to provide accurate solution, in an acceptable finite time and in an economically way. Another related aspect refers to the modeling approach considered by hydrologists, i.e.: the Saint-Venant equations to surface free flow [4]. This methodology just allows the acquisition of discharge and liquid level, but the velocity flow and pressure distribution remain unknowns. As a consequence the required continuity between the hydraulic head and the liquid total pressure at the interface is not taken into account, leading

to a serious drawback – the poor accuracy properties of the approximated solutions.

In this work we present a mass conservative Navier-Stokes formulation for the incompressible free surface flow, combined with a Darcy – Richard's approach for the subsurface porous media flow. Coupling between these two media flows are physically reproduced incorporating the interface conditions related to: pressure balance; velocity field continuity; and a shear stress jump condition [5] between both domains. Using the above assumptions we first derive the continuous model, considering that the porous media solid skeleton is incompressible, and that a water-air non-saturated fluid flows through this medium. Then, a Galerkin finite element procedure is used to approximate diffusion dominated coupled flows. After that, a stabilized Petrov-Galerkin finite element model based on the CAU method, as proposed by Galeão et al [8], is presented to accurately approximate the full transient coupled problem, when high velocity free water flows occur. For both models, a low order P1P0 polynomial interpolation [6, 7], (velocity field (linear – continuous) and (constant – discontinuous) pressure/hydraulic head field), is employed. Non-linear terms are treated by a Picard quasi-Newton scheme, and GMRES solvers are used for the numerical solution of the resulting algebraic system of equations.

Finally, some numerical experiments are performed, to highlight the main features of the proposed models.

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