A closer look at the numerical errors of the immersed boundary method

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ABSTRACT

The immersed boundary method is an often used method in case of moving and complicated geometries. Many examples can be found of application of these methods.

An important point regarding immersed boundary methods concerns the error analysis. Validations have been of course been shown together with the applications showing the viability of immersed boundary methods. Nevertheless, some more analysis of the error and its sources would be welcome. It could lead to a more proper understanding and improvement of existing methods.

Errors occur because of several reasons. One reason is the interpolation procedure close to the interface. Another is the fact that usually a pressure-correction method is used. The resulting pressure correction is applied after using the immersed boundary method in the solution of the momentum equations. This introduces an additional error, for instance there can be a non-zero flow through the immersed boundary.

A step towards the error analysis could be the separation of the two problems, interpolation and pressure correction. This separation is not always easy. However, for a rectangular geometry this can be done. To this end a variant of the method used by Verzicco [1] is used, which shown in figure 1 together with the original method used by him. As can be seen, the way the new variant [2] is applied is the same as for a standard solver, there is no additional interpolation, it uses one point less inside the fluid. This also holds for the region near a rectangular corner.

The results we aim to show in the mini-symposium are comparisons for Large-Eddy simulations with the original Verzicco's method, its new variant and a standard solver, using an iterative Poisson solver, plus a standard solver using Verzicco's treatment of the boundary. Geometries are channels with and without obstacles, for instance a street canyon with pollutant dispersion, see figure 2.

The key point is that we have several variants which can be compared which differ only with respect to one item, namely boundary interpolation or pressure correction. Results can be shown for quantities very close to the surface (the first grid cell near the boundary) and for quantities depending on first



Figure 1: Left: Cartesian method of Verzicco. After solution of the momentum equations as if no wall is there, velocity components do not extrapolate to 0 on the fictitious wall. The velocity component nearest to the wall is adapted so that it does. Right: Variant of immersed boundary method. Velocities are calculated as if no wall is there. After this, the normal velocity on the horizontal cell wall is put to 0, and the contribution from the shear stress which was applied at the horizontal wall (which is then not correct) is subtracted and replaced by the stress that would exist in case of a real wall. The last stress is in discretised form equal to $\frac{U}{1/2dz}$

grid cell variables like heat flux. They continue on results shown in [3] and shed some light on questions which part of the error (interpolation, pressure correction) is the dominant one and the effect of variations in the boundary treatment.



Figure 2: Left: a geometry tested with several variants of boundary treatment. Flow over a street canyon with a pollution source. Right: a result for the RMS of the pollution concentration in the first grid cell near the left boundary. Three methods tested, Verzicco's original method (verz), the new variant (stress) and results from a standard solver (standard).

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