

## ESTIMATION OF INLET VELOCITY BASED ON POD INVERSE SOLUTION OF THE UNSTEADY HEAT TRANSFER PROBLEM

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**Key Words:** *Inverse problem, heat transfer, proper orthogonal decomposition.*

### ABSTRACT

The solution of the unsteady state inverse convection-diffusion heat transfer problem in the system with the potential fluid flow and phase change is presented and discussed in the paper. The aim is the estimation of the fluid inlet velocity.

Heat transfer processes with phase change occurring during artificial ground freezing is investigated. Ground freezing is often employed as the first step of many activities, which are concerned with drilling of shafts, construction of tunnels or bridges, as well as prevention against the escape of liquid contaminants from waste sites. The flow of groundwater, whose velocity is very low, can seriously influence such a process [1]. To determine optimal freezing parameters, a numerical model of the process should include the groundwater flow. One of the crucial input parameters in such a model is the inlet velocity of groundwater. The aim of the study is to estimate this velocity using the additional information coming from measurements of the temperature during unsteady process at a selected set of points.

The considered transient heat transfer problem in the freezing ground with flowing groundwater is described by two coupled equations: transient energy transfer and steady state groundwater flow. The transient component of the latter may be neglected as the process of freezing and the movement of the interface is very slow. The groundwater flow in water-saturated ground treated as a porous medium is modelled employing Darcy's law. The porous matrix and groundwater are assumed to be in local thermal equilibrium. The problem is formulated in the dimensionless variable system. In this way, the estimation of the inlet velocity based on temperature measurements reduces the problem to the evaluation of the Peclet number, playing a role of a coefficient in energy equation. This number is determined making use of measurements of the dimensionless temperatures in the computational domain..

Similar problem of retrieving inlet velocity was analysed in [2], however in the case of the steady state process and for single freezing/heating tube only. In the present investigations, the transient inverse heat transfer problem is solved. The method called as POD-RBF, which employs Proper Orthogonal Decomposition and the Radial Basis

Functions, is applied for this purpose. This method used previously to solve steady state inverse problems, e.g. in [3], serves two important features: reduction of number of DOFs necessary to describe the spatial distribution of the temperature and filtering out the noise.

At the first step of the proposed technique, each retrieved parameter is sampled in the vicinity of the predicted solution at several levels. In the next step, a sequence of transient direct problems is solved for the sampled values of unknown parameters. The resulting potential and temperature fields are obtained using an in house FEM code. The sequence of so achieved discrete fields corresponding to different time instants and sampled values of retrieved parameters, is further referred to as snapshots. The set of snapshots is the input of POD analysis yielding a truncated set of orthogonal, uncorrelated modes of the temperature field. The low-order approximate solution of the forward problem is expressed as a sum of POD modes multiplied by unknown amplitudes. These amplitudes are expressed next as a linear combination of the Radial Basis Functions (RBFs) requiring that the POD-RBF approximation is exact for all snapshots. The argument of these functions is the distance between two vectors whose components are time and the retrieved parameters. The first vector gathers the current values of time and retrieved parameters, while the second one collects values that were used to generate the snapshots. Obtained in this way low order POD-RBF approximation of the temperature field as a function of time and retrieved parameters is employed to fit the simulated and the measured temperatures.

The 2D examples presented in the paper are related with ground freezing process performed by a single tube with freezing medium and by system of freezing tubes arranged in a single circle. The problem of retrieving a constant value of the Peclet number and its linearly time dependent value is investigated.

## REFERENCES

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