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Computing interior eigenvalues of a generalized complex-symmetric pencil arising from the modeling of resonant MEMS systems

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ABSTRACT

Understanding the behavior of high frequency MEMS resonators is of high interest due to their potential applications as on-chip, small sized, energy efficient, frequency references or signal processing filters. Their performance is determined by the quality factor Q, which measures the sharpness of a resonance peak and is inversely proportional to the amount of damping in the system. A high Q is desired, thus damping must be minimized in the design of the device. Q can also be expressed in terms of the complex-valued eigenvalues of the system ω as $Q = \frac{|\omega|}{2\mathrm{Imag}[\omega]}$. This expression enables one to formulate the problem of computing Q as a matrix eigenvalue problem, where the matrices are obtained from a numerical discretization of the governing equations of motion of the system.

Among the many damping mechanisms that exist, it has been observed that acoustic loss is a prominent source in the design of high frequency resonators. In this mechanism, the motion of the resonator couples with the underlying substrate through its anchors, sending outgoing propagating waves which never return, resulting in energy loss. To model this phenomenon numerically, one must apply proper wave absorbing boundary conditions at the computational domain boundary, in order to mimic the infinite domain behavior. The technology of Perfectly Matched Layers(PML)[1] is selected here to serve this purpose.

The difficulty of solving for eigenvalues of the system above are due to the application of the PML, the sensitivity of acoustic loss to the system design, and the modes of interest. Application of the PML results in complex-valued symmetric non-Hermitian mass and stiffness matrices and quite fine meshes are required for accurate computations. The magnitude of acoustic loss is sensitive to the geometric design of the device, and in certain cases 2-D models of the actual 3-D device may not serve as accurate models to estimate damping. The eigenvalues corresponding to modes of interest may not be at the exterior of the spectrum. Thus to obtain accurate solutions, one requires a method to compute interior generalized eigenvalues of large-scale complex-valued symmetric matrices through parallel computing.

The solution method chosen is the Jacobi-Davidson QZ(JDQZ) algorithm[2] combined with a geometric multigrid preconditioner to solve the correction equations. Compared to Krylov subspace-based

methods such as shift-and-invert Arnoldi, the Jacobi-Davidson schemes does not require machine precision accurate linear solves to expand the projection subspace, which is ideal when the linear systems are large and iterative solves are the only choice. To decrease the number of iterations in the iterative linear solve, geometric multigrid, one of the few methods that is known to scale effectively to large mechanical problems is used. The different levels in the multigrid scheme are constructed automatically by parameterizing the block-generated mesh by a few length scale variables such as the approximate distance between adjacent nodes. The prolongators between two levels are then computed by evaluating nodal points in the finer level mesh at the coarser level mesh shape functions. The smoother used is the standard Gauss-Seidel scheme. To enhance JDQZs speed of convergence to the desired eigenvalue and mode, the initial shift and starting vector is obtained from an approximation constructed from the coarser grids in the multigrid hierarchy. Once the coarse problem eigenvalue problem is solved, the eigenvalue is used as the initial shift, and the corresponding eigenvector is prolongated with the multigrid prolongator for the starting vector. The algorithm is implemented in the MEMS simulation software HiQLab[3] combined with the well established PETSc[4] library built on top of MPI.

A 20[μm] diameter polysilicon disk resonator is simulated and a radial contour mode at 1.18[GHz] is computed. Fig.1 shows a 2D slice of the resonator through the center post and substrate, where the colors shows the x and z direction displacements of the mode. One can see wave propagation through the substrate and the damped behavior in the PML region at the boundary of the computational domain, as well as the z direction bending type of motion in the disk. The convergence of Q and frequency with respect to the discretization size is given in Fig.2, for linear, quadratic, and cubic finite elements. The computations are conducted on 16 processors. For the maximum size problem of 6 million degrees of freedom, only 2 outer JD iterations and under 80 inner GMRES iterations per JD iteration is required, showing the effectiveness of the proposed method.

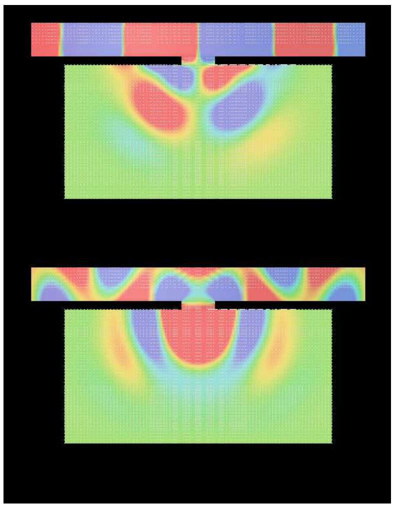

Figure 1: Eigen mode at 1.18[GHz], Top:x disp, Bottom: z disp

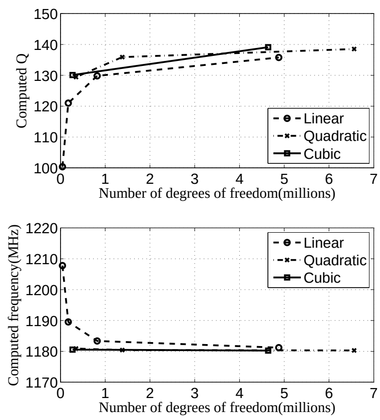

Figure 2: Convergence of Q and frequency

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