Discontinuity-Capturing and the Variational Multiscale Method

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ABSTRACT

In his 1954 dissertation, Godunov proved that monotone linear numerical schemes for solving partial differential equations can be at most first-order accurate. Consequently, a number of nonlinear numerical schemes have been proposed with the hopes of obtaining monotonicity. In the finite element community, residual-based artificial viscosities have traditionally been added to a stabilized formulation. The design of these discontinuity-capturing terms has been largely motivated by entropy analysis, and their implementation is often more of an art than a science.

In this talk, we discuss an alternative approach to the design of discontinuity-capturing terms through the framework of the variational multiscale (VMS) method. In the VMS method, the solution u is decomposed into a coarse-scale component $\bar{u}$ and a fine-scale component u'. The scale splitting is defined by means of an optimality condition. To ensure a monotone solution, we define our optimality condition to be the minimization of $\|u-\bar{u}\|^2_{H^1_0}$ subject to a total variation constraint on $\bar{u}$. This definition leads to a variational formulation of a character altogether different than other VMS schemes. In fact, it leads to a multiscale finite element formulation with a new pair of discontinuity-capturing terms. We will discuss the relationship between this new VMS formulation with more traditional residual-based artificial viscosities and the design of new stabilized methods based on this construct.

REFERENCES

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