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Non-conformal XFEM approach and Spider XFEM : error estimates and numerical experiments

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ABSTRACT

Numerical experiments showed many advantages of XFEM when modeling cracked materials (see [1,2,3]). A first attempt to obtain an optimal convergence with XFEM was the surface enrichment method which is expensive (see [2,3]). Then, a surface global enrichment method using a cutoff function was introduced in [3]. It was observed that the transition layer between the enriched area an the non-enriched one increases the committed error (see [3,4,5]).

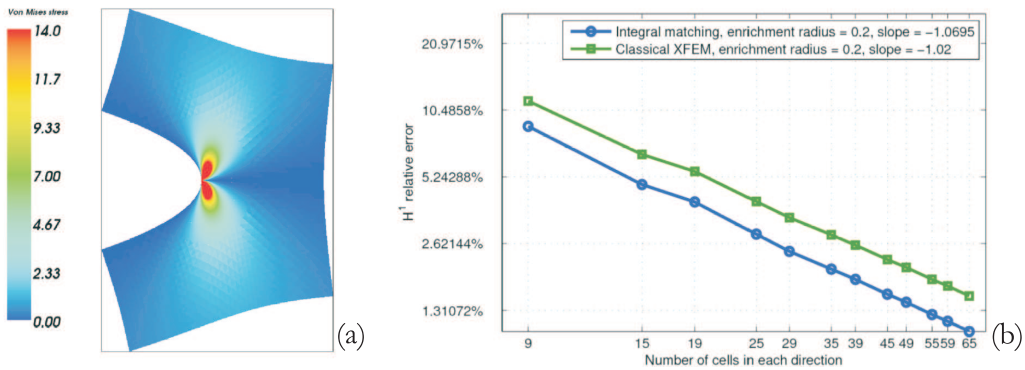

FIG. 1 – *(a) Von Mises stress, (b) Energy norm relative error with respect to h^{-1} in log. scales*

In the first variant we consider, the transition layer is removed and replaced by a *mortar* type integral matching condition. This non-conformal strategy reduces significantly the number of degrees of freedom with respect to the classical XFEM and enhances the accuracy of the approximation with respect to the classical surface enrichment method (see fig.1(b)). Moreover, an optimal error estimate is proved (see [4,6]).

In other respects, the use of XFEM type methods becomes very expensive or impossible when the singularity is too complicated or partially unknown. This is the case e.g. for bimaterial interface cracks (see [7]) or composite plate cracks. We introduce a new variant, the so-called *Spider XFEM* (see [8]), which allows to deal with such cases. Denoting r and θ the polar coordinates with respect to the crack tip, we consider a singular enrichment where the dependency in θ of the asymptotic displacement is approximated by a classical FEM defined over a circular mesh (*the spider mesh*, see fig.2(a)). The method reduces the number of enrichment functions, which decreases the computational cost for complex problems. It can be used also when the exact singularity is partially unknown. A mathematical result of optimal convergence is obtained for this approach under some condition on the spider mesh parameter (see [4,8]). This is illustrated by the numerical tests in fig.2(b).

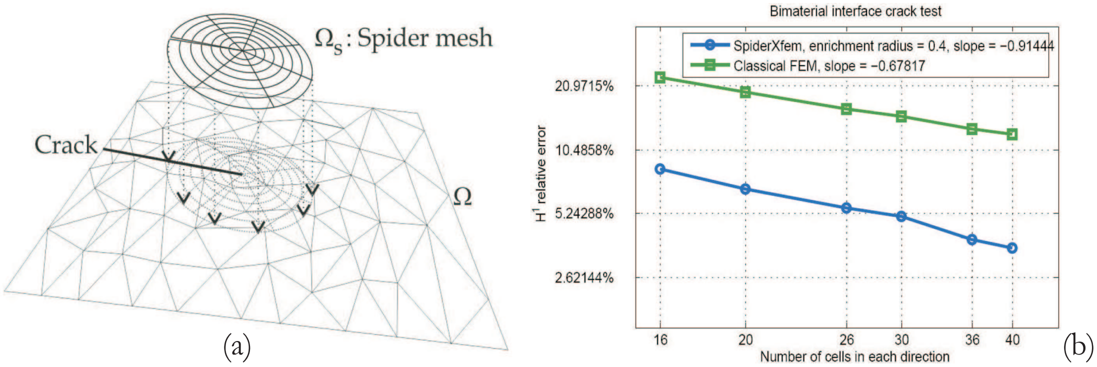

FIG. 2 – *(a) The resulting mesh, (b) Energy norm relative error with respect to* h^{-1} *in log. scales*

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