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# LOCAL METHOD FOR IDENTIFICATION OF PARAMETERS IN MATHEMATICAL MODEL OF VESTIBULAR AFFERENT NEURON

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## ABSTRACT

A modified Hodgkin - Huxley system of three nonlinear ordinary differential equation of the first order is considered as a mathematical model for the action potential generation of the vestibular afferent neuron [1]:

$$\frac{dV}{dt} = I_{apl} - h\, m_{\infty}^{3}(V)\bar{g}_{Na}(V - V_{Na}) - n^4 h\, \bar{g}_K (V - V_K) - \bar{g}_l (V - V_L), \quad (1)$$

$$\frac{dn}{dt} = a_n(V)(1-n) - b_n(V)n, \quad (2)$$

$$\frac{dh}{dt} = a_h(V)(1-h) - b_h(V)h, \quad (3)$$

where *V(t)* is a potential function, *n(t)* is activation variable and *h(t)* is inactivation variable, $0 \le n, h \le 1$; $V_{Na}, V_K$ are normalised potentials of the balance of sodium *Na*, potassium *K* correspondently; $V_L$ is the potential of the filtration balance; $\bar{g}_{Na}$, $\bar{g}_K$, $\bar{g}_L$ are maximal conductivities of sodium, potassium and of filtration correspondently; $I_{apl}$ is control current, function $m_{\infty}(V)$ is calculated on the base of the experimental data.

The direct problem for system (1) - (3) consists in its solution for given functions $a_n(V)$, $b_n(V)$, $a_h(V)$, $b_h(V)$ and values $V_0$, $n_0$ $h_0$ according to the Cauchy conditions $V(0) = V_0$, $n(0) = n_0$, $h(0) = h_0$.

The inverse problem consists in recuperation of variable coefficients $a_n$, $b_n$, $a_h$, $b_h$ using data obtained by "voltage clamp recordings" experiments. The traditional empirical approximations of these functions can be presented by rational-exponential formulas with constant parameters [1]. Identification of these parameters requires solution of the system of three complicated non linear equations, corresponding to (1) - (3).

The local method for identification of two unknown functional parameters in simplified Hodgkin - Huxley system was proposed in [2]. Here we present the development of the Local method for the system (1) - (3).

We suppose that the input data of the voltage clamp recordings experiments can be consider as $N$ measured values $V_i = V(t_i)$ of the potential function on the greed $\{t_i\}$, *i=1,…,N,* of the time variable with nodes at close range. The proposing approximations $\tilde{a}_n, \tilde{b}_n\ \tilde{a}_h, \tilde{b}_h$ for functions $a_n, b_n,\ a_h, b_h$ are presented by piecewise-constant expansions with unknown coefficients $\{a_i\}$, $\{b_i\}$**:**

$$\tilde{a}_n(V) = \sum_{i=1}^{N-1} a_i s_i(V)\,,\quad \tilde{a}_h(V) = \sum_{i=1}^{N-1} b_i s_i(V) \qquad (4)$$

$$\tilde{b}_n(V) = \sum_{i=1}^{N-1} a_{i+N-1} s_i(V)\,,\ \tilde{b}_h(V) = \sum_{i=1}^{N-1} b_{i+N-1} s_i(V)\,. \qquad (5)$$

Here piecewise-constant basic functions $s_i(V)$ are constructed on the auxiliary greed $\{\bar{V}_i\}$, *i=1,…,N* such that: $\min_i \{V_i\} = \bar{V}_1 \le \bar{V}_2 \le \ldots \le \bar{V}_N = \max_i \{V_i\}$.

The proposing Local method for the reconstruction of the coefficients $\{a_i\}$, $\{b_i\}$ consists in the next parts: 1) to calculate the approximation for the derivative $dV/dt$ in the points $\{t_i\}$ by divided differences; 2) to calculate approximate values $n_i$ for $n(t_i)$ and $h_i$ for $h(t_i)$ from equation (1); 3) to calculate the approximations $n_i'$, $h_i'$ for the derivatives $dn/dt$, $dh/dt$ in the points $t_i$ by divided differences; 4) to calculate for every index *i=1,…,N-1* the coefficients $a_i$ $a_{i+N-1}$, $b_i$ $b_{i+N-1}$ as the solution of the two **local** collocation schemes

$$n_i' = (1-n_i)a_i - n_i a_{i+N-1}, \qquad (6)$$

$$n_{i+1}' = (1-n_{i+1})a_i - n_{i+1}a_{i+N-1}. \qquad (7)$$

$$h_i' = (1-h_i)b_i - h_i b_{i+N-1}, \qquad (8)$$

$$h_{i+1}' = (1-h_{i+1})b_i - h_{i+1}b_{i+N-1}. \qquad (9)$$

Systems of linear algebraic equations (6), (7) and (8), (9) are independent. Solutions of each exist and is unique if functions *n(t), h(t)* are not constant.

Local method gives us really the explicit approximate solution of the inverse coefficient problem and therefore it is simpler in the numerical realization than traditional schemes, which require solution of some non linear equations. Proposed method is realized as stable algorithms using spline regularization for numerical differentiation. Computer software in MATLAB system is constructed and justified by numerical experiments with simulated and real experimental data.

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