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An implicit energetic approach of 3D crack growth under fatigue loading and residual stresses

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ABSTRACT

Reliable and efficient strategies for simulation of cracked structures are of great interest in e.g. aerospace industry. To avoid catastrophic failure of damaged key components, reliable tools are required in order to precisely predict crack growth resulting from repeated loadings. Whereas two-dimensional crack propagation is now commonplace in industry, the rapid increase of computing power also provides the ability to treat three-dimensional cracked structures and stimulates the development of algorithms for fatigue crack growth for three-dimensional configurations.

The approach presented herein deals with a linear elastic structure subjected to both mechanical loading and imposed temperature field. The initial state is characterised by an *a priori* know residual stress field. An energetic fracture mechanics approach is invoked in order to simulate the evolution of the structure, under the assumption of a planar crack path. For a stable growth, the method consists in seeking the minimum of the total structure energy $E = J + D$ (where J is the mechanical potential energy and J and a fracture energy D), with the respect to the structure displacement field $\underline{u}$ and all possible admissible crack geometries Γ [1]:

$$E\left(\underline{u}, \Gamma\right) = \min_{\left(\underline{u}^{\star}, \Gamma^{\star}\right)} E\left(\underline{u}^{\star}, \Gamma^{\star}\right)$$

In a general case, solving such problem can be very costly [2]. However, simpler procedures occur when a single, connected crack is considered together with a fracture energy of Griffith type [3,4].

Thus, the minimisation problem can be solved by an incremental-iterative method. For each load increment, the final displacement $\underline{u}$ and crack configuration Γ are found by iteratively solving the stationarity equations associated to the above minimization problem. Each iteration therefore consists of finding u from

$$E_{,\underline{u}}\left(\underline{u}\right) = 0 \Leftrightarrow D_{,\underline{u}}\left(\underline{u}\right) + J_{,\underline{u}}\left(\underline{u}\right) = 0,$$

which is a usual linear elastic analysis. Then, the crack surface Γ is updated by solving the stationarity equation

$$E_{,\Gamma}\left(\Gamma\right) = 0 \Leftrightarrow D_{,\Gamma}\left(\Gamma\right) + J_{,\Gamma}\left(\Gamma\right) = 0$$

using a Newton-Raphson method. This step requires the first-order and second-order domain derivatives of the elastic potential energy and the fracture energy, applied to domain perturbations that correspond to infinitesimal crack growth, in order to find the crack increment from the linearized equation

$$E^{(2)}\left(\Delta\Gamma,\underline{\theta}^{\star}\right) = -E^{(1)}\left(\underline{\theta}^{\star}\right), \forall\, \underline{\theta}^{\star} \text{ adm.},$$

where $E^{(1)}$ and $E^{(2)}$ are respectively the first-order and second-order derivatives of the total structure energy with respect to the crack front advance and $\underline{\theta}^{\star}$ are trial crack growth velocities that are used as test functions.

Analytical expressions of the energy derivatives $E^{(1)}$ and $E^{(2)}$ are established using the G-θ method initiated by Destuynder and Djaoua [5], and in particular extended to cases where given residual stress, body load or temperature fields occur in the region of the crack front. New terms, not shown here for brevity, then arise in $E^{(1)}$ and $E^{(2)}$ as a result of these added effects.

According to the Griffith assumption, the fracture energy D is defined in terms of the density of surface energy released by crack growth. For monotonic loading and stable growth, this density is classically taken constant and equal to a critical value G_c. For fatigue problems with the crack growth governed by a Paris-type law which is the main focus of this communication, the incremental fracture energy D is defined so as to recover the Paris law from the first-order stationarity condition for E :

$$D = \int_{\gamma} \frac{mC\Delta N}{m+1} \left(\frac{\Delta a(s)}{C\,\Delta N}\right)^{1+\frac{1}{m}} \mathrm{d}s$$

where $\Delta a(s)$ denotes the local crack advance at a generic point of the current crack front γ with arclength coordinate s.

This energetic approach has been implemented within the ZeBuLoN finite element environment (www.nwnumerics.com) with a robust remeshing process. The ability of the proposed algorithm to compute planar fatigue crack growth in the presence of residual stresses or thermal loadings is demonstrated through numerical experiments (which will be shown during the conference) on a thermally loaded penny-shaped crack, for which an analytical solution is available for comparaison purposes.

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