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A new semi-analytical method for the construction of auxiliary fields in the interaction integrals of 3-D LEFM

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ABSTRACT

The study of crack propagation is since a long time a crucial issue in research and industry ; there are everincreasing needs for robust and predictive simulation tools for cracking in three-dimensional, mixed-mode conditions. Global approaches (G) such as the G-θ method introduced by Destuynder [1], which consists in evaluating the derivative of the potential energy of the solution under a virtual crack extension modelled by a transformation velocity field θ, are very efficient because of their robustness. In particular, unlike local methods (K), they can be performed on meshes without special crack-front elements. However, the G-θ method evaluate the energy release rate G, which has for certain applications the disadvantage of mixing the fracture modes, i.e. the stress intensity factors (SIFs). Separation evaluation of each SIF is often useful or necessary, e.g. for the implementation of additional criteria predicting future kinking or curving of the crack(s).

The SIFs can be uncoupled within a global, G-θ based computational framework, by means of the concept of *interaction integral* [2]. Interaction integrals (or M-θ integrals) are defined as the bilinear contribution arising in the expression of the G-θ domain integral applied to the superposition of two states, namely the solution (u, σ[u]) solution for crack and loading configuration being considered, and a judiciously defined auxiliary state (v, σ[v]). Specially, the latter is chosen so as to correspond to a given distribution of SIFs along the crack front.

Interaction integrals, like the G-θ integral, thus provide the basis of a post-processing of the FEM solution, allowing separate computation of the three SIF distributions associated with the solution. The success of this method depends on the validity of the auxiliary fields chosen, which must satisfy the compatibility, equilibrium, and (linear-elastic) constitutive relations. For 2-D problems involving straight cracks, the auxiliary fields can simply be chosen in terms of the Westergaard solutions. However, analytical solutions for cracks of arbitrary geometry are not known, especially in the 3-D case. To circumvent this difficulty, several methods for constructing auxiliary fields are proposed. Some [2] use for each point of the crack front the Westergaard solutions written in curvilinear coordinates ajusted to the crack geometry, sometimes with complementary terms arising from the curvature of the crack

front and faces. Others use the analytical solutions for a penny-shaped crack, adapted to the curvature at each point of the crack front [3]. All these approaches entail a degree of approximation, ultimately caused by the curvature of the crack or the crack front. For example, the Westergaard solutions written in curvilinear coordinates do not satisfy the equilibrium and compatibility equations.

In this work, a new method for constructing auxiliary fields for the extraction of the stress intensity factors is proposed. This method guarantees (winthin discretization errors) the satisfaction of the (equilibrium, compatibility and constitutive) field equations for arbitrary crack geometries. The basic idea consists in using a integral representation formula of a displacement field [4] written in terms of a displacement jump ϕ_i throught the crack selected so as to satisfy one of the conditions on auxiliary SIFs:

$$v\left(x\right)=\int_{\widehat{S}}\phi_i\left(y\right)T_i^k\left(x,y\right)\ dS \tag{1}$$

where T_i^k denotes the traction vector associated with the free-space elastostatic Green's tensor, i.e. the Kelvin solution. Any displacement of the form (1) satisfies by construction the field equations of linear elasticity. In addition, the support $\widehat{S}\in S$ of this displacement jump is confined to a small area around the point of crack front at wich the stress intensity factors are sought, in order to minimize the computational burden entailled by this postprocessing. The displacement jump in the integral representation being chosen a priori (so as to conform with a desired distribution of SIF on the crack front), the auxiliary fields thus defined develops non zero tractions on the crack faces. It is thus necessary to adapt accordingly the expression of the interaction integral derived from the G-θ approach.

The main computational steps for evaluating the interactions integral using the present approach are as follows :

- Calculation of the tractions associated to auxiliary displacement (1) on portion $\widehat{S}\in S$ where the transformation velocity field θ of the G-θ method is non-zero;
- Calculation of a new contribution to the interaction integral having the form of a surface integral on $\widehat{S}$ involving the previously conputed tractions;
- Calculation of $\nabla v\left(x\right)$ in the support $\widehat{\Omega}\in\Omega$ of the vicinity field. The regions $\widehat{S}$ and $\widehat{\Omega}$ being local (surface and volume neighbouhoods of the crack front point) this procedure is computationally economical. This algorithm provides a means for computing, via a by postprocessing of the solution for a given crack configuration, the stress intensity factors for a crack of arbitrary 3-D geometry. In the proposed communication, we will present the above-outlined computational algorithm and numerical results designed to assess, by means of comparisons with the known analytical solutions of a penny-shaped crack, the accuracy and stability of the proposed approach.

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