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Pointwise Identification of Elastic Properties in Nonlinear Hyperelastic Membranes

*** Jia Lu, Xuefeng Zhao**

Department of Mechanical and Industrial Engineering
The University of Iowa
Iowa City, IA 52242, USA
jia-lu@uiowa.edu

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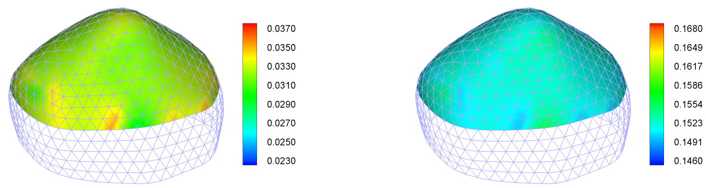
ABSTRACT

We introduce an experimental method capable of delineating the pointwise elastic property in hyperelastic membranes. The method is a non-trivial generalization of the finite strain inflation test by Rivlin and Sanders [1]. The original inflation test was limited to axisymmetric membrane structures; in our development it is extended to general convex membranes without any geometric symmetry. The cornerstone of this new experimental method is a non-conventional approach of stress analysis, the finite element inverse elastostatics method. This method takes as the input a deformed configuration of an elastic material body and the corresponding load, and determines the stress distribution in the given deformed state by way of finding the initial stress-free configuration. For finitely deforming membranes the inverse approach has a unique advantage, that is, it can effectively compute the membrane wall stress without knowing the realistic material constitutive equation. This is possible because the wall stress in a pressurized membrane structure is determined by the applied load and the deformed geometry. By taking the deformed configuration as the input, the inverse method can sharply capitalize on the static determinate nature of the wall stress [2]. The inverse approach of stress analysis, together with a suitable method for strain data acquisition, enables us to obtain pointwise stress-strain data independently, and thus to delineate the distributive properties in the membrane wall.

In this presentation, we will discuss the theoretical underpins of the method and present the numerical and physical experiments we conducted to validate the method. In the numerical experiments, we computed a series of deformed configurations of a sac-shaped membrane under various pressure levels. The surface was initially constructed from medical images of a cerebral aneurysm which does not have any particular geometric symmetry. A hyperelastic model (*Model A*) with assumed elastic parameters was utilized in generating the deformed configurations. The deformation history was used as input data for parameter identification; the localized stress strain data were fit to the *Model A* and another distinct model (*Model B*). In both cases satisfactory regression results were obtained. As an illustration, the distribution of the identified elastic parameters of the *Model B* are shown in Figure 1.

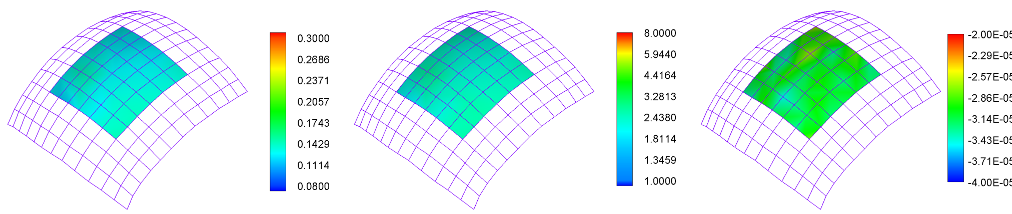
Experimental validation was conducted through a finite inflation tests on a rubber balloon. An optical motion tracking system was developed based on a photo-based 3-D reconstruction technique. A finite element mesh was drawn over a selected region in the balloon surface, and the balloon was pressurized

to several pressures. After reconstructing the surface meshes, we applied the method to identify the distributive properties by fitting the pointwise stress-strain data to the Ogden model. The material was identified to be intrinsically homogeneous, excluding the local variations of the wall thickness. Figure 2 illustrates the distribution of the identified elastic parameters. A comparison between the model stress-strain curves and the experimental data at a selected point is provided in Figure 3.



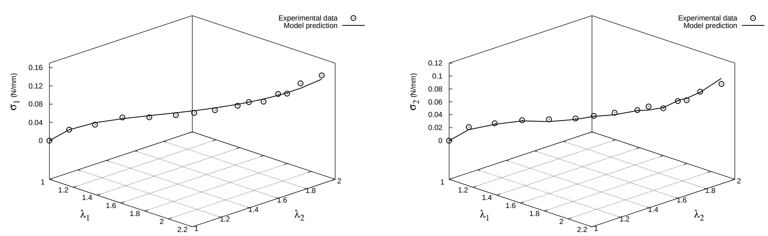
(a) : ν_1 (b) : ν_2

Figure 1: Identified elasticity parameters of *Model B* which contains two elastic parameters ν_1 and ν_2.



(a) : μ_1 (b) : μ_2 (c) : μ_3

Figure 2: Identified Ogden model parameters μ_1, μ_2, and μ_3.



(a) : σ_1 (b) : σ_2

Figure 3: Comparison between the “experimental” principal stresses σ_1 and σ_2 and the model predictions.

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