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STOCHASTIC REDUCED ORDER MODELING TECHNIQUES FOR DESIGN UNDER UNCERTAINTY

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ABSTRACT

Over the past decade, stochastic projection methods (SPMs), in particular approaches based on polynomial chaos expansion (PCE), have evolved into mature tools for predicting the response of engineering systems governed by stochastic partial differential equations (SPDEs) [1]. Both intrusive methods requiring modifications to the assembly of operators of the discretized SPDEs and non-intrusive methods allowing the re-use of existing PDE solvers in a black-box fashion have become popular. In particular, PCE methods have been successfully applied to a broad range of structural and multi-physics problems. From a design optimization point of view, SPMs are an appealing alternative to traditional reliability methods as they do not lead to nested optimization problems and can be used to efficiently evaluate both design criteria characterizing the robustness and criteria quantifying the reliability of a system.

However, the computational costs of SPMs increases significantly with the number of random input parameters defining the stochastic variability of a system and the size of the numerical mechanical model describing the deterministic response. For example, using an intrusive PCE approach, assuming a moderate size mechanical model with 10^4 degrees of freedom and applying a third order polynomial expansion of the stochastic response for three random input parameters, the number of unknowns in the PCE problem is close to $2 \cdot 10^5$. Similarly, the number of samples needed for non-intrusive approaches grows rapidly with the number of random parameters and order of polynomial expansion. The numerical burden of SPMs can be alleviated, for example, through the application of iterative (parallel) solvers. However, this approach is often insufficient in the context of design optimization requiring the repeated evaluation of the stochastic model as well as the computation of the design sensitivities.

A promising alternative for significantly decreasing the computational burden of SPMs is the integration of reduced order modeling techniques projecting the solution of a full order model into an appropriately chosen lower order subspace [2,3]. Reduced order models (ROMs), such as Krylov subspace methods, Padé approximation, modal analysis, and Proper Orthogonal Decomposition, have been frequently used as fast re-analysis methods. ROMs approximate, for example, the transfer function of a forced-vibration problem over a range of excitation frequencies and static or transient response of linear and nonlinear

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problems over a range of parameters for control, design optimization and stochastic analysis purposes. Recently, ROMs have also been explored for reducing the computational costs of PCE methods [4,5,6]. In the non-intrusive case, the ROM simply is used instead of the full order model in a black-box fashion. In the intrusive case, model reduction can be applied to the deterministic mechanical model and then the reduced response is expanded by PCE. We refer to methods following this scheme as ROM-PCE approaches. Alternatively, model reduction can be directly applied to the PCE model derived from the full order mechanical model. Such methods are labeled PCE-ROM approaches.

Building on the promising results of using ROMs in PCE methods, we present a general computational framework for design optimization under uncertainty using reduced order modeling techniques. Single and multi-point ROM methods enriched by gradient information are used to approximate the full-order mechanical model over a range of random and design parameters. A trust-region strategy is proposed for controlling the adaptation and recalibration of the ROMs. We provide an overview of efficient reduced order modeling techniques focusing on linear static and transient problem. Different concepts for integrating ROMs into intrusive PCE formulations are presented and analyzed focusing on ROM-PCE methods. The associated design sensitivity equations are presented. The accuracy and numerical efficiency of the intrusive approaches are compared against non-intrusive ROM-based PCE methods. Numerical results for linear elastic composite shell structure show that both approaches lead to the same accuracy with comparable numerical efforts. Furthermore, the results indicate that single-point ROMs enhanced by basis vectors representing the derivatives of the system response with respect to random parameters are sufficient for stochastic analysis purposes. In contrast, an accumulated multi-point ROM strategy is needed for approximating the system response in the space of the design parameters as changes in system response due to design changes are typically large.

The numerical efficiency of the proposed design optimization framework is studied with shape optimization of a spherical shell structure subject to stochastic imperfections in its thickness distribution. The shape changes are defined via a design element method and the stochastic field is parameterized by a Karhunen-Loeve approach. The design problem is to maximize the mean stiffness with constraints on the stress-based failure probabilities. The numerical results show that the optimization process employing full- and reduced order models lead to nearly the same solution. However, the proposed integration of ROMs into SPMs allow for a reduction of the computational costs by almost two orders of magnitude.

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