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A NUMERICAL SCHEME TO MOVING BOUNDARIES OF POROUS MEDIUM FLOW IN OIL RESERVOIR

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ABSTRACT

Let us consider a problem of two immiscible fluids, say *water* and *oil*, in an oil reservoir. Specially we are interested in the case where water is injected into the reservoir and water pushes oil. In this case the moving boundary, which separates the region where water penetrates from the oil region, is unstable (fingering phenomenon). Such a problem is described by

$$s_t + \nabla \cdot [v f(s)] - \nabla \cdot [f(s)\phi(s)\nabla s] = 0\,; \tag{1}$$

$$\nabla \cdot v = 0\,; \tag{2}$$

$$v = -(\lambda(s)\nabla p - \phi(s)\nabla s), \tag{3}$$

where $s = s(x,t)$, $v = v(x,t)$, and $p = p(x,t)$ are the saturation of water, velocity of water and oil, and the pressure of water, respectively. Known functions $f(s)$, $\lambda(s)$, and $\phi(s)$ are determined by the ratio of viscosities, relative permeabilities and capillary pressure. Typically,

$$f(s) = \frac{s^2}{\lambda(s)}, \quad \lambda(s) = s^2 + \frac{1}{\mu}(1-s)^2, \quad \phi(s) = \frac{c(1-s)^2}{\mu}, \tag{4}$$

where $\mu > 0$ is the ratio of viscosities of oil and water, and $c > 0$ shows a magnitude of the capillary pressure (see [4], for example). We note that the moving boundary is defined by $\Gamma(t) = \overline{\{x \mid s(x,t) > 0\}} \cap \overline{\{x \mid s(x,t) = 0\}}$.

The aim in this paper is to propose a numerical scheme to (1)–(3). Our numerical scheme is based on the TCD (Threshold Competition Dynamics) method [1], which is a methodology to obtain an approximate solution to some class of reaction–diffusion system with *singular* reaction terms.

Our strategy is as follows:

1. Find a reaction–diffusion system which approximates (1) when $k \to \infty$, where k is a parameter appearing in the singular reaction terms of the system.
2. Use the TCD method to obtain a numerical solution to the reaction–diffusion system with $k = \infty$.

As a result, a numerical solution to (1)–(3) is obtained. Merits of our numerical scheme are summarized as

- No artificial parameter is used ;
- Moving boundary can be captured naturally ;
- Computational costs are comparative low ;
- Algorithm is simple.

We note that a similar numerical scheme to the porous medium equation

$$u_t = \Delta u^m \quad (m > 1 \text{ is constant}) \tag{5}$$

is already proposed in [3], and that the diffusion process in (1) is formally described by (5) with $m = 3$. It should also be noted that a numerical scheme to Stefan problem is constructed under same strategy (see [2]).

Finally we demonstrate a numerical simulation in Figure 1. The system (1)–(3) is solved in $(x, y) \in \Omega \equiv (0, 1)^2$ and $0 < t \leq 2$. The parameters are $\mu = 20$ and $c = 10^{-5}$. We use the following boundary condition:

$$\begin{cases} s(0, y, t) = 1, & p(0, y, t) = 1, \qquad 0 < y < 1,\ 0 < t \leq 2, \\ s(1, y, t) = 0, & p(1, y, t) = 0, \qquad 0 < y < 1,\ 0 < t \leq 2, \\ s_y(x, y, t) = 0, & p_y(x, y, t) = 1, \qquad 0 < x < 1,\ y \in \{0, 1\},\ 0 < t \leq 2. \end{cases} \tag{6}$$

This condition means that water is injected from the left edge $\{0\} \times (0, 1)$ and that oil is recovered at the right edge $\{1\} \times (0, 1)$. The up and down edges $(0, 1) \times \{0, 1\}$ are assumed Neumann boundaries.

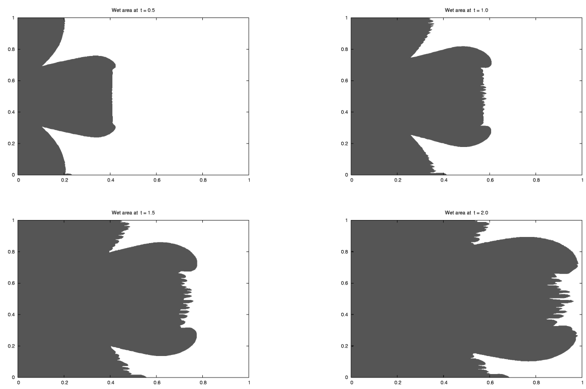

Fig. 1: Numerical simulation in 2-dimensional domain $\Omega = (0, 1)^2$. Numerical moving boundaries at $t = 0.5, 1, 1.5, 2$ are shown. Water is penetrated (i.e., $s > 0$) in gray area. Fingering phenomenon can be observed.

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