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A hyperelastic-visco-elasto-plastic-damage model for rubber-like solids

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ABSTRACT

Rubber materials useed in engineering fields are known to possess many complicated properties such as Mullins' effect, rate-dependency, hardening and eternal deformation. Although the phenomena have been modeled by hyperelasticity with damage[1], viscoelasticity[2], viscoplasticity[3] and mixed models of these three types[4],[5], their performance for reproducing real behavior seem to be insufficient, especially under cyclic deformation in several large strain levels.

This work presents a new constitutive model for rubber-like solids with emphasis on reproducing hysteretic behavior in different strain levels. The model employs a Hyperelastic-Damage(:HD) element for expressing direction of stress evolution, and multiple Visco-Elasto-Plastic (:VEP) elements are connected to HD element in parallel for reproducing hysteretic behavior. In the formualtion, material logarithmic strain tensor $\mathbf{H} := 1/2 \ln \mathbf{C}$ ($\mathbf{C}$: right-Cauchy-Green tensor) is used as a measure of relative deformation, while its stress-conjugate is approximated by rotated Kirhhoff stress tensor $\mathbf{T} := \mathbf{R}^T \cdot \boldsymbol{\tau} \cdot \mathbf{R}$ ($\boldsymbol{\tau}$: Kirhhoff stress tensor, $\mathbf{R}$: rotation tensor) for simpicity[6]. Then, the total free-energy of the model is described as:

$$\Phi(\mathbf{H}, d, \mathbf{H}_k^{(e)}) = \Phi_0(\mathbf{H}, d) + \sum_{k=1}^{n} \Phi_k(\mathbf{H}, \mathbf{H}_k^{(e)}) \tag{1}$$

where Φ_0 and Φ_k are the free-energies of HD element and VEP elements, respectively, d (scalar) and $\mathbf{H}_k^{(e)}$ (tensor) are internal variables for inelastic behavior. As usual, Φ_0 is divided into isochoric and volumetric parts as follows.

$$\Phi_0(\mathbf{H}, d) = (1-d) \cdot \bar{\psi}_0(\mathbf{G}) + U_0(\Delta_1) \text{ with } \mathbf{G} := \mathrm{dev}[\mathbf{H}] \text{ and } \Delta_1 := \mathrm{tr}(\mathbf{H}) \tag{2}$$

Detailed forms of $\bar{\psi}_0$, U_0 and an evolution equation of d are:

$$\bar{\psi}_0 = \sum_{k=1}^{N_0} \frac{\mu_k}{\alpha_k} (e^{\alpha_k \bar{\zeta}_1} + e^{\alpha_k \bar{\zeta}_2} + e^{\alpha_k \bar{\zeta}_3} - 3), \; U_0 = \frac{\kappa_0}{2} (e^{\Delta_1} - 1)^2 \tag{3}$$

$$d(t) := r_0 \tanh\left[\frac{\Xi(t) - \bar{\psi}_0(\mathbf{G}(t))}{m_0 + \beta_0 \cdot \Xi(t)} \right], \; \Xi(t) := \max_{0 \le s \le t} \bar{\psi}_0(\mathbf{G}(s)) \tag{4}$$

where $(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\zeta}_3)$ is the pricipal values of $\mathbf{G}$, and μ_k, α_k $(k = 1, 2, \cdots, N_0)$, κ_0, r_0, m_0 and β_0 are parameters of HD element.

The key features of the model are forms of VEP elements, which only express isochoric behavior and employ an unified visco-elastic/elasto-plastic/visco-plastic model. In

addtion, free-energies of VEP elements are extended by introducing a new function $\Lambda_k(\mathbf{G})$, which we call 'Hardening-Rate', in $\Phi_k(\mathbf{H},\mathbf{H}_k^{(e)})$ as:

$$\Phi_k(\mathbf{H},\mathbf{H}_k^{(e)}) = \psi_k(\mathbf{G}_k^{(e)})\cdot\Lambda_k(\mathbf{G}) \text{ with } \mathbf{G}_k^{(e)} := \text{dev}[\mathbf{H}_k^{(e)}] \tag{5}$$

Detailed forms of ψ_k, Λ_k and evolution equations of $\mathbf{H}_k^{(e)}$ are as follows.

$$\psi_k := \mu_k^{(e)}\left|\mathbf{G}_k^{(e)}\right|^2,\ \Lambda_k := 1 + \frac{e^{m_k\bar{\zeta}_1} + e^{m_k\bar{\zeta}_2} + e^{m_k\bar{\zeta}_3} - 3}{h_k^{m_k}} \tag{6}$$

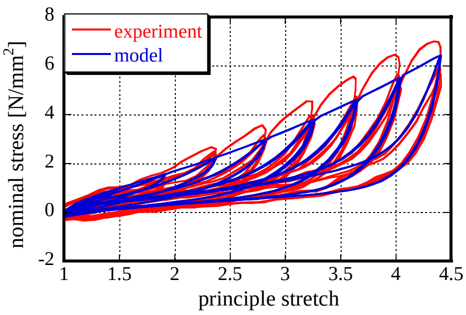
$$\dot{\mathbf{H}} - \dot{\mathbf{H}}_k^{(e)} = \left[(1-\theta_k)\frac{\bar{\varepsilon}_k}{\lambda_k} + \theta_k\left(\frac{3}{2}\left|\dot{\mathbf{G}}\right|^2\right)^{\frac{1}{2}}\right]\sqrt{\frac{2}{3}}\left(\frac{3\left|\hat{\mathbf{T}}_k^{(1)}\right|^2}{2(\bar{\sigma}_k^{(Y)})^2}\right)^{\frac{N_k}{2}}\frac{\hat{\mathbf{T}}_k^{(1)}}{\left|\hat{\mathbf{T}}_k^{(1)}\right|} \text{ with } \hat{\mathbf{T}}_k^{(1)} = \frac{\partial\psi_k}{\partial\mathbf{H}_k^{(e)}} \tag{7}$$

where $\mu_k^{(e)}$, m_k, θ_k, $\bar{\varepsilon}_k$, λ_k and N_k are parameters of VEP elements.

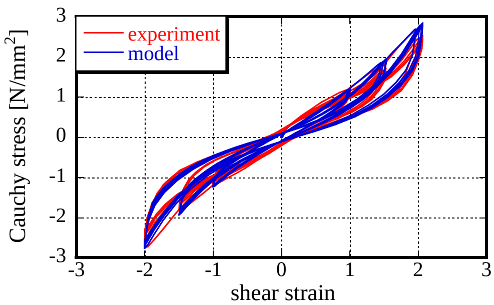
Finally, the stress-strain relation is obtained under condition that the following approximated Clausius-Duhem inequality[6] is satisfied in arbitrary deformation processes.

$$\mathfrak{D} := \boldsymbol{\tau} : \mathbf{d} - \dot{\Phi} \approx \mathbf{T} : \dot{\mathbf{H}} - \dot{\Phi} \geq 0 \quad (\mathbf{d} : \text{defomation rate tensor}) \tag{8}$$

Fig.2 shows comparions of the propsed model with two VEP elements and experimental results of simple tension and simple shear tests, and it is found that the model can accurately reproduce complex hysteretic behavior under cyclic deformation in several strain levels of different deformation types.



(a) uniaxial tension test



(b) simple shear test

Fig.2 Comparison of the propsoed model with experimental results

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