

## LOCAL ADHESIVE CONTACT LAW: FINITE ELEMENT IMPLEMENTATION AND CONSEQUENCES OF LARGE DEFORMATION

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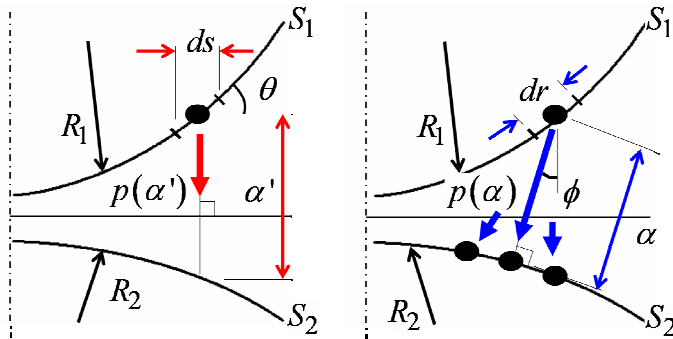
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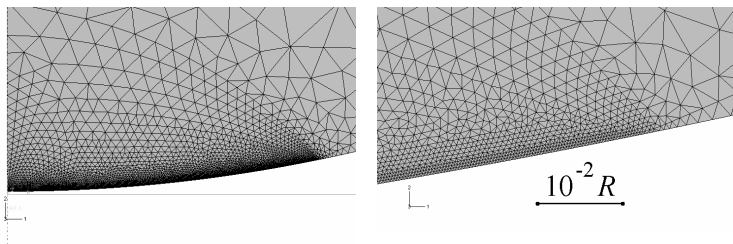
### ABSTRACT

In this work we focus on the development of a finite element (FE) model to analyze the adhesive contact between two elastic spheres. The hallmark of the FE model is the use of a local contact model where the surface traction at a point on the interacting surfaces is dependent on a single parameter – the distance measured along the normal to the companion surface. The resultant load on the spheres is computed using the current deformed area. The local contact model is used to study the implications of the Derjaguin approximation and the assumption of small deformation (small strains and rotations) in the existing models. Using the Derjaguin approximation, the distance between any two points on the interacting surfaces is computed as being perpendicular to plane of first contact – the tangent plane passing through the points of first contact. The small deformation assumption treats the interacting surfaces as slightly deformed so that the resultant load on the spheres can be computed using the reference projected area on the tangent plane.



**Figure 1:**  
 Illustration of contact traction with (left) and without Derjaguin approximation.

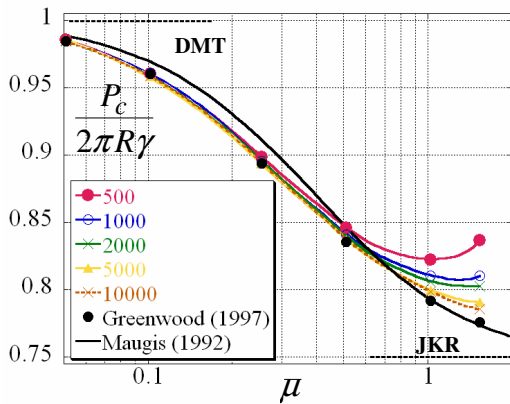
In the FE model, a biased mesh with dense meshing close to the contact area is used to accurately capture the peak tensile traction. The smallest elements have their side length of the order of the interatomic equilibrium distance. Both linear and quadratic interpolation elements are used to ensure convergence of the solution.



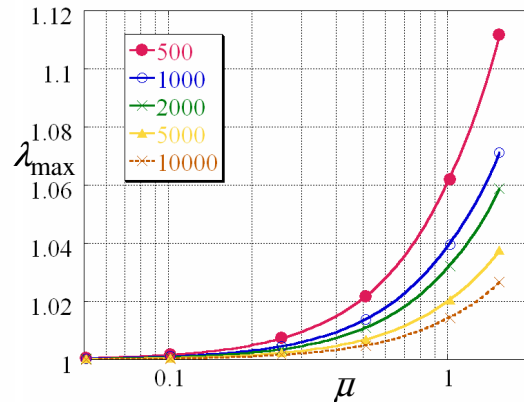
**Figure 2.:**  
 Details of a section of the FE mesh.

In the solution process, the ill-conditioned stiffness matrix during the unstable jump to contact and pull-off is circumvented using viscous damping and those solutions are later verified by the arc-length method. In addition to the standard checks used in the iterative solvers, manual verification of the magnitude of the peak tensile traction and the ratio of the energy damped to the strain energy of the spheres in the stable portion is done to obtain accurate solutions. The results from FE analysis using Lennard-Jones type (LJ) contact laws are compared with the values from existing adhesion models at different values of modified Tabor parameter  $\bar{\mu}$ . The modified Tabor parameter is obtained by approximating the LJ force law as a triangular force law with a range  $\alpha_r$  instead of using the interatomic equilibrium distance as the range as in the original Tabor parameter.

The pull-off load values for small stiff spheres with weak adhesion (small values of  $\bar{\mu}$ ) compare well with the existing models. For large values of  $\bar{\mu}$ , the FE results predict an increasing trend of the normalized pull-off load instead of approaching the JKR limit and the magnitude of the pull-off load at was dependent of the effective radius of the spheres  $R$ . This discrepancy is attributed to the failure of the existing models to account for the stretch of the interacting surfaces due to the radial component of the surface traction. The magnitude of the radial component of traction is dependent on the angle mismatch between the normal to the deformed surface and the surface traction at that point. In larger spheres, the angle is small leading to small stretches. In general the larger pull-off load is due to the integration of the compressive tractions over a smaller area (stretch ratio  $\lambda < 1$ ) and the tensile tractions over a larger area ( $\lambda > 1$ )



**Figure 3:** Pull-off load values ( $P_c$ ) from the FE analysis with the existing models at different values of  $R$  and  $\bar{\mu}$ .



**Figure 4:** Maximum stretch ratio.

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