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Multiscale Mortar Mixed Methods For Elliptic And Parabolic Problems Arising From Darcy Flows

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ABSTRACT

In this paper, we consider second order linear elliptic and parabolic equations that model single phase Darcy flows in porous media.

First, we study multiscale mortar mixed finite element discretizations for model elliptic problems introduced by Arbogast *et al.* [1]. This approach is based on domain decomposition theory and mortar finite elements [2]. In this method, flux continuity is imposed via a mortar finite element space on a coarse grid scale, while the equations in the coarse elements (or subdomains) are discretized on a fine grid scale.

We extend the method to treat slightly compressible Darcy flows in porous media in [3]. Parallel numerical simulations on some multiscale benchmark problems are given to show the efficiency and effectiveness of the method.

We consider an interface problem involving only the mortar pressure which arises from the mortar mixed formulation. The interface formulation is useful in deriving a bound on the error in the mortar space. Moreover, it is the basis for implementation of parallel domain decomposition methods. However, in the previous work [1], the mortar pressure error was measured in a semi-norm arising from the interface formulation. It applies to the case where the model problem is symmetric positive definite. It should be noted that in many interesting applications, it is important to be able to treat more general problems including nonsymmetric tensor coefficients and/or convection-diffusion equations. Moreover, in the nonlinear case, the use of the superposition principle is not applicable for the error analysis. Therefore, we introduce a new approach for treating interface problem and prove various stability estimates based on inf-sup conditions related to the mortar pressure variable [4]. Optimal fine scale convergence is obtained by an appropriate choice of mortar grid and polynomial space of approximation.

Finally, we discuss recent results treating convection-diffusion equations and slightly compressible Darcy flows using this new approach.

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